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COMPARATIVE STUDY OF PRODUCTIVITY OF BOTTOM AND EDGE WATER DRIVE RESERVOIRS

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Abstract: *Objective of this study is to develop edge water drive reservoir and bottom water drive reservoir which will be used to carry out a comparative study of productivity of the reservoirs. The aim is to know which one is more suitable in terms of productivity and this was carried out by developing a model using source function and Newman product rule. Numerical method was used to compute dimensionless pressure and dimensionless pressure derivative. Correlation coefficient was determined to know if there is any correlation that exit between the models. The value shows that there is, hence R2 was determined to know which one has higher productivity. The results show that Bottom water drive is more suitable in terms of productivity.*
Key words: *Source function, Pressure derivative, Bottom water.*

Uporedna studija produktivnosti donjih i rubnih rezervoara za vodu. *Cilj ove studije je razviti rubni rezervoar za vodu i donji rezervoar za vodu koji će se koristiti za sprovođenje uporedne studije o produktivnosti rezervoara. Cilj je znati koji je pogodniji u pogledu produktivnosti, a to je izvedeno razvojem modela koristeći funkciju izvora i Nevman-ovog pravila proizvoda. Numerička metoda je korišćena za izračunavanje bezdimenzionalnog pritiska rezervoara i bezdimenzionalnog pritiska derivata. Koeficijent korelacije određen je kako bi se znalo da li postoji neka korelacija izlaza između modela. Vrjednost pokazuje da postoji, zbog toga je određen koeficijent R2 kako bi se znalo koji ima veću produktivnost. Rezultati pokazuju da je donji vodni pogon pogodniji u pogledu produktivnosti.*
Ključne reči: *Izvorna funkcija, derivat pod pritiskom, voda na dnu.*

1. INTRODUCTION

Water move from hydrocarbons out of the reservoir, into the wellbore and up to surface. Water drive reservoirs can have bottom water drive or Edgewater drive. In a bottom water-drive reservoir, water is located beneath the oil accumulation, while in an edge water-drive reservoir, water is located only on the edges of the reservoir [1]. The presence of edge water has both advantages and disadvantages for the improvement of offshore heavy oil reservoir. On one hand, edge water invasion could supplement the reservoir pressure which is in favor of the oil production, while on the other hand, the asymmetrical invasion of the edge water could result in the quick increase of water cut for production wells and the consequent low oil recovery efficiency [2]. A common problem in edge-water drive reservoirs is oil bypassing by aquifer water. The encroaching water from the adjacent aquifer overtakes oil phase and leaves a significant volume of trapped residual oil behind. Early arrival of these water fingers causes pre-mature water production that leads to well abandonment. One solution to the problem is creating a barrier against the encroaching water [3]

The objective of this study is to develop models of different water drive (edge and bottom drives) which will be used to carry out a comparative study of productive of the reservoirs. The aim is to know which one is more suitable in terms of productivity.

2. MATERIALS AND METHODS

The following steps are taken in these works.

1. Boundary condition will be chosen for each axis.
2. Appropriate source function for each axis will be selected [4] (Gringarten*, et al*., 1973).
3. Newman product rule will be applied to arrive at the pressure expression.
4. The derivative of pressure expression will be carried out.
5. Determination of correlation coefficient.
6. Determination of R^2.

2.1Reservoir Physical Model

Reservoir physical models under study are shown in Figure 1 and Figure 2. They shows edge water and bottom water with horizontal well.

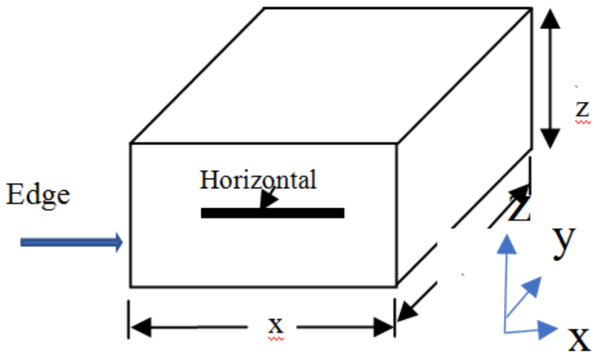

Fig. 1. Reservoir subject to Edge water

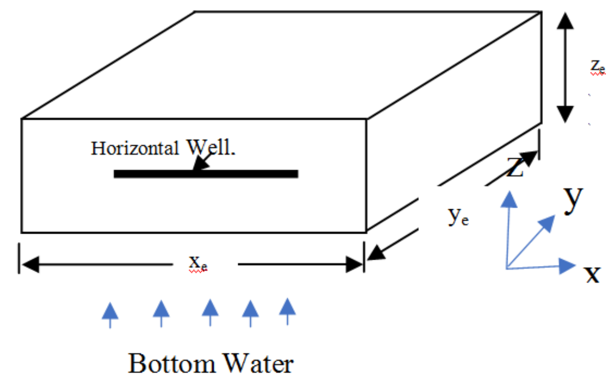

Fig. 2. Bottom water and sealed at x and y axes

Source functions for Figure 1 are carefully chosen from basic instantaneous source functions table for x, y and z axis's given as xi(x), v(y) , v(z) [5].

$$(x_{D}, t_{D}) = \frac{8}{\pi} \sum_{n-1}^{\infty} \frac{1}{2n-1} exp\left[-\frac{(2n-1)^2 \pi^2 t_D}{X_{eD}^2}\right] \quad (1)$$
$$* Cos\frac{(2n-1)\pi x_D}{2x_{eD}} sin\frac{(2n-1)\pi x_D}{x_{eD}} sin\frac{(2n-1)\pi x_{WD}}{x_{eD}}$$

$$s(y_{D,}t_D) = \frac{1}{y_D} \quad (2)$$
$$*\left\{1+2\sum_{n=1}^{\infty} exp\left[-\frac{m^2 n^2 t_D}{y_D}\right] * Cos\frac{m\pi y_{wD}}{y_D} Cos\frac{m\pi y_D}{y_D}\right\}$$

$$s(z_{D,}t_D) = \frac{1}{h_D} \quad (3)$$
$$*\left\{1+2\sum_{n=1}^{\infty} exp\left[-\frac{l^2 n^2 t_D}{h_D}\right] Cos\frac{l\pi z_{wD}}{h_D} Cos\frac{l\pi h_D}{h_D}\right\}$$

$$P_D = 2\pi h_D \int_0^{t_D} s(x_D, t_D) s(y_D, t_D) s(z_D, t_D) \partial t_D \quad (4)$$

Substituting equation 1, 2 and 3 in 4 we have Equation 5 as the pressure distribution for reservoir sealed at x and y axes and subjected by edge water.

$$P_D = 16 h_D \int_0^{t_D} \begin{matrix} \sum_{n-1}^{\infty} \frac{1}{2n-1} exp\left[-\frac{(2n-1)^2 \pi^2 t_D}{x_{eD}^2}\right] \\ * Cos\frac{(2n-1)\pi x_D}{2x_{eD}} sin\frac{(2n-1)\pi x_D}{x_{eD}} sin\frac{(2n-1)\pi x_{wD}}{x_{eD}} \\ * \frac{1}{y_D}\left\{1+2\sum_{n=1}^{\infty} exp\left[-\frac{m^2 n^2 t_D}{y_D}\right] * Cos\frac{m\pi y_{wD}}{y_D} Cos\frac{m\pi y_D}{y_D}\right\} \\ * \frac{1}{h_D}\left\{1+2\sum_{n=1}^{\infty} exp\left[-\frac{l^2 n^2 t_D}{y_D}\right] * Cos\frac{l\pi z_{wD}}{h_D} Cos\frac{l\pi h_D}{h_D}\right\} \end{matrix} \quad (5)$$

Equation 5 is the dimensionless pressure for the reservoir system. The dimensionless pressure derivative given as

$$P'_D = t_D \frac{\partial P_D}{\partial t_D} \quad (6)$$

Using Equation 6, Equation 7 is derived as

$$P'_D = 16 h_D \left[\begin{matrix} \sum_{n-1}^{\infty} \frac{1}{2n-1} exp\left[-\frac{(2n-1)^2 \pi^2 t_D}{x_{eD}^2}\right] \\ * Cos\frac{(2n-1)\pi x_D}{2x_{eD}} sin\frac{(2n-1)\pi x_D}{x_{eD}} sin\frac{(2n-1)\pi x_{wD}}{x_{eD}} \\ * \frac{1}{y_D}\left\{1+2\sum_{n=1}^{\infty} exp\left[-\frac{m^2 n^2 t_D}{y_D}\right]\right\} * \\ Cos\frac{m\pi y_{wD}}{y_D} Cos\frac{m\pi y_D}{y_D} \frac{1}{h_D}\left\{1+2\sum_{n=1}^{\infty} exp\left[-\frac{l^2 n^2 t_D}{y_D}\right]\right\} * \\ Cos\frac{l\pi z_{wD}}{h_D} Cos\frac{l\pi h_D}{h_D} (7) \end{matrix}\right] \quad (7)$$

For Figure 2, the source functions are also selected from source Table and they are as follows:

Equation 8, 9 and 10 are the source functions for the three axes in dimensionless form.

$$s(x_D, t_D) = \frac{2}{x_{eD}} \{1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \begin{matrix} \frac{1}{n} exp\left[-\frac{n^2 x_D^2 t_D}{x_{eD}^2}\right] \\ Sin\frac{n\pi}{x_{eD}} Cos\frac{n\pi x_{wD}}{x_{eD}} Cos\frac{n\pi x_D}{x_{eD}} \end{matrix} \} \quad (8)$$

$$s(y_D, t_D) = \frac{2}{y_D} \{2 \sum_{n=1}^{\infty} \begin{matrix} \frac{1}{n} exp\left[-\frac{m^2 x_D^2 t_D}{y_D}\right] \\ Cos\frac{m\pi y_{wD}}{y_D} Cos\frac{m\pi y_D}{y_D} \end{matrix} \} \quad (9)$$

$$s(z_D, t_D) = \frac{2}{h_D} \{1 + 2 \sum_{n=1}^{\infty} \begin{matrix} \frac{1}{n} exp\left[-\frac{l^2 \pi^2 t_D}{h_D}\right] * \\ Cos\frac{l\pi z_{wD}}{h_D} Cos\frac{l\pi h_D}{h_D} \end{matrix} \} \quad (10)$$

Putting equation 8, 9 and 10 in 4 we have Equation 11 as the pressure distribution for reservoir sealed at x and y axis's and subjected by bottom water.

$$P_D = 2\pi h_D \int_0^{t_D} \frac{2}{x_{eD}} \{1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \begin{matrix} \frac{1}{n} exp\left[-\frac{n^2 x_D^2 t_D}{x_{eD}^2}\right] \\ Sin\frac{n\pi}{x_{eD}} Cos\frac{n\pi x_{wD}}{x_{eD}} Cos\frac{n\pi x_D}{x_{eD}} \end{matrix} \} * \quad (11)$$
$$\frac{2}{y_D} \{2 \sum_{n=1}^{\infty} \begin{matrix} \frac{1}{n} exp\left[-\frac{m^2 x_D^2 t_D}{y_D}\right] \\ Cos\frac{m\pi y_{wD}}{y_D} Cos\frac{m\pi y_D}{y_D} \end{matrix} \} * \frac{2}{h_D} \{1 + 2 \sum_{n=1}^{\infty} \begin{matrix} \frac{1}{n} exp\left[-\frac{l^2 \pi^2 t_D}{h_D}\right] * \\ Cos\frac{l\pi z_{wD}}{h_D} Cos\frac{l\pi h_D}{h_D} \end{matrix} \}$$

Using Equation 6, Equation 12 is derived as

$$P_D = 2\pi h_D \left[\begin{array}{l} \frac{2}{x_{eD}}\left\{1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\frac{n^2 x^2_D t_D}{x^2_{eD}}\right] Sin\frac{n\pi}{x_{eD}} Cos\frac{n\pi x_{wD}}{x_{eD}} Cos\frac{n\pi x_D}{x_{eD}}\right\} * \\ \frac{2}{y_D}\left\{2\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\frac{m^2 x^2_D t_D}{y_D}\right] Cos\frac{m\pi y_{wD}}{y_D} Cos\frac{m\pi y_D}{y_D}\right\} * \\ \frac{2}{h_D}\left\{1+2\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\frac{l^2\pi^2 t_D}{h_D}\right] * Cos\frac{l\pi z_{wD}}{h_D} Cos\frac{l\pi h_D}{h_D}\right\} \end{array} \right] \quad (12)$$

Calculating the Pearson Correlation Coefficient(R) With Dimensionless Pressure of Edge Water and Bottom Water Reservoirs

Pearson Correlation Coefficient(r) is given as [6]

$$r = \frac{\sum P_{D(Bottom)} P_{D(Edge)} - \frac{\left(\sum P_{D(Bottom)}\right)\left(\sum P_{D(Edge)}\right)}{N}}{\sqrt{\sum\left[P_{(Bottom)}\right]^2 \frac{\left(\sum P_{(Bottom)}\right)^2}{N} * \sum\left[P_{(Edge)}\right]^2 \frac{\left(\sum P_{(Edge)}\right)^2}{N}}} \quad (13)$$

Where

$P_{D(Bottom)}$ =Dimensionless Pressure for bottom water drive reservoir

$P_{D(Edge)}$ = Dimensionless Pressure for edge water drive reservoir

N =Size of samples

The reservoir and well parameters used for both models are shown in Table 1

XeD	yeD	XD	XwD	Yd	ywD	zwD	Zd	Hd
6	2	0.25	0.28	0.74	0.74	0.55	0.55	6

Table 1. Reservoir and Well Parameters

3. RESULTS AND DISCUSION

The results from the two models are presented below. The pressure and pressure derivative distribution for Edge Water Drive Reservoir are presented in Table 2.

t_D	P_D	P'_D
0.001	183.188	2.59E-07
1.00E-02	1.75E+02	2.43E-04
0.1	117.117	0.142673
1	65.78674	52.59596
10	64.88	-965.366
100	64.88	1.10E-06
1000	64.88	-2.00E-112
10000	64.88	0

Table 2. Pressure and Pressure Derivative Distribution for Edge Water Drive Reservoir.

The results show that as production continue the pressure decreases and became stable. This stableness is as a result of the fluid encountering external boundary and it is at late time. While that of pressure derivative we have negative values, this show that water is produced during oil production as shown in Figure 1.

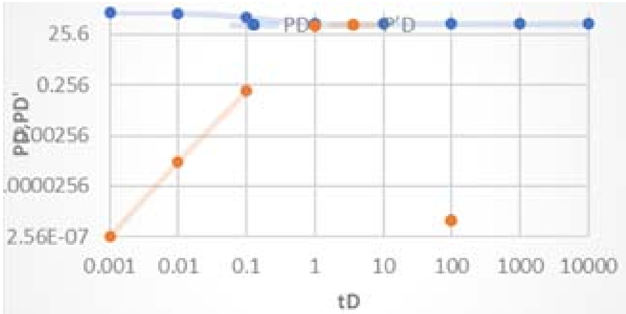

Fig. 1. P_D and P_D' for edge water drive Reservoir

Table 3 is pressure and pressure derivative distribution of bottom water drive reservoir. The productivity here is higher compared to that of edge water drive reservoir. As the production continue the pressure decreases and it became stable from $t_D = 10$ to $t_D = 10000$. An active bottom-water drive reservoir would improve oil recovery, reduce water production due to coning, delay water breakthrough time, and pre-identify wells that are candidates to excessive water production [7]. While the pressure derivative decreases to zero as shown in Figure 2.

Td	P_D	P'_D
0.001	392.1135	6.37E-04
1.00E-02	3.77E+02	5.41E-03
0.1	275.744	0.016596
1	149.3497	0.057895
10	138.7593	0.049
100	138.7593	9.25E-12
1000	138.7593	5.30E-118
10000	138.7593	0

Table 3. Pressure and Pressure Derivative Distribution for Bottom Water Drive Reservoir

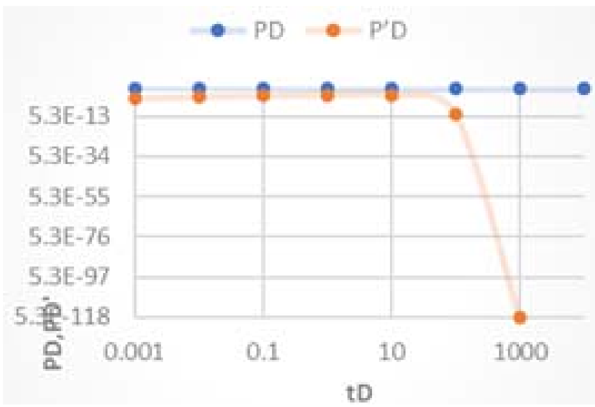

Fig. 2. PD and PD' for Bottom water drive Reservoir

t_D	$P_{D(y_{wD}=0.74)}$	$P'_{D(y_{wD}=0.74)}$	$P_{D(ywD=1.2)}$	$P'_{D(XwD=1.2)}$	$P_{D(ywD=0.32)}$	$P'_{D(YwD=0.32)}$
0.001	183.188	2.59E-07	183.1881	1.49E-07	183.1881	3.34E-07
1.00E-02	1.75E+02	2.43E-04	1.75E+02	1.41E-04	1.75E+02	3.12E-04
0.1	117.117	0.142673	117.117	0.092529	117.117	0.1767
1	65.78674	52.59596	65.7867	50.1652	65.78674	54.2457
10	64.88	-965.366	64.888	-965.366	64.888	-965.366
100	64.88	1.10E-06	64.888	1.10E-06	64.888	1.10E-06
1000	64.88	-2.00E-112	64.888	-1.97E-112	64.888	-1.97E-112
10000	64.88	0	64.888	0	64.888	0

Table 4 Effect of Y_{wD} on P_D and P'_D For Edge Water Drive Reservoir

The location of well in Y direction does not affect the pressure and pressure derivative for both edge and bottom water drive reservoirs as shown in Table 3 and 4 respectively.

In order to carryout comparative study of the two models, there is need to determine if there is any correlation between these models by using equation 11.

Using equation 11

Where

$P_{D(Bottom)}$ =1749.244

$\sum P_{D(Edge)}$ =800.6117

$(\sum PD(Bottom))^2$ =471238.7

$(\sum PD(Edge))^2$ =99064.79

$\sum$PD(Bottom) PD(Edge)=215935.8

Substituting the values, we have

$$r = \frac{215935.8 - \frac{1749244 * 800.6117}{N}}{\sqrt{\left[1749244^2 - \frac{471238.7}{N}\right] * \left[[800.6117^2 - \frac{99064.79}{N}\right]}} = 0.9969374$$

Therefore r= **0.9969374**

This mean that there is correlation between dimensionless pressure of bottom and dimensionless pressure of edge water drive reservoir. Haven established that there is correlation, there is need to determine the reliability and effective of the two models determining the R squared (R^2) of both models. This was done by plotting the values of P_D and P'_D against t_D and a trendline is drawn and R^2 is determined as displaced on the graphs in Figure 3 through to Figure 5. R^2 for both P_D and P'_D for bottom water drive reservoir is higher when compared with edge water drive. Hence the model supported by bottom water drive more reliable and effective.

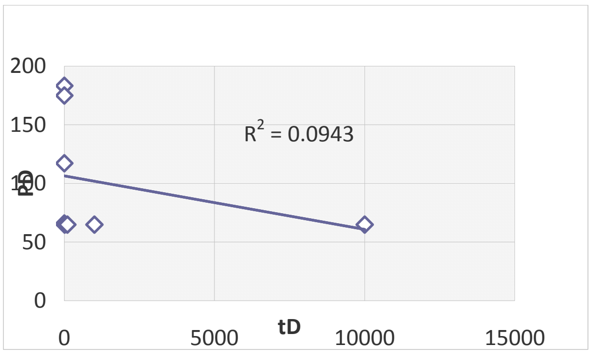

Fig. 3. R^2 of P_D For Edge Water Drive Reservoir

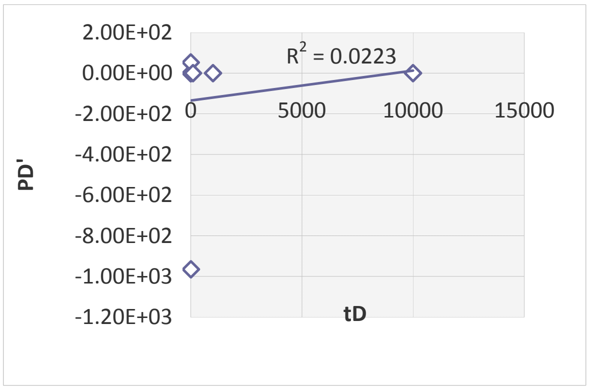

Fig. 4. R^2 of P_D' For Edge Water Drive Reservoir

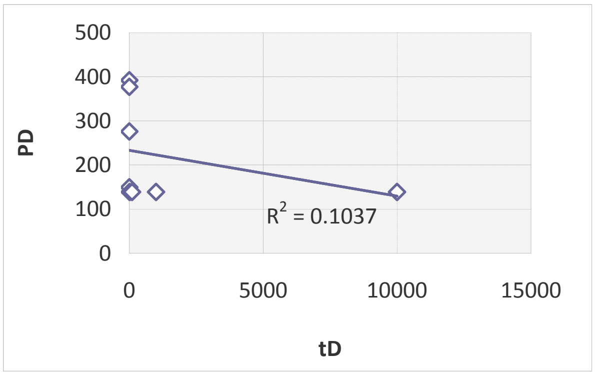

Fig. 5. R^2 Of P_D For Bottom Water Drive Reservoir

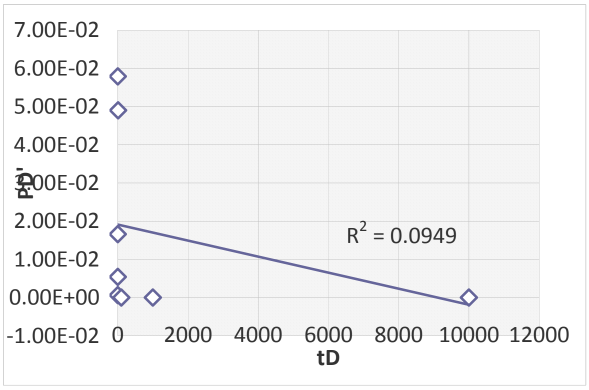

Fig. 6. R^2 of P_D' For Bottom Water Drive Reservoir

4.CONCLUSION

This study revealed that there is

1. Correlation between the productivity from the two

models,
2. Productivity from bottom water drive is higher,
3. There is no water production unlike that of edge water drive and
4. R^2 of reservoir subjected by bottom water drive is higher than that of edge water drive.
5. Bottom water drive reservoir is more suitable.

Nomenclature

Yw_D = Well coordinate in y-direction
Yw_D = Well coordinate in y-direction
Yw_D = Well coordinate in y-direction
Ye_D = External dimensionless distance along y-axis
Xw_D = Well coordinate in x-direction
t_D= Dimensionless time
P'_D = Dimensionless Pressure derivative
P_D= Dimensionless Pressure
h_D= Dimensionless height
X_D= arbitrary dimensionless distance along the x-axis
Y_D = arbitrary dimensionless distance along the y-axis
Z_D = arbitrary dimensionless distance along the z-axis
Zw_D = Well coordinate in z-direction

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