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Bányai, T.

SENSITIVITY ANALYSIS OF WAGNER-WHITIN ALGORITHM

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Abstract: *Material requirement planning (MRP) plays an important role in the efficiency improvement of manufacturing companies. The MRP solutions of enterprise resource planning (ERP) systems are influenced by both technological and logistics parameters, but using additional algorithms, like Wagner-Whitin or Silver-Meal heuristics, it is possible to take more parameters into consideration. These heuristics can optimise the results of MRP, especially from ordering, warehousing and transportation costs point of view. Within the frame of this article the impact of process parameters on the result of Wagner-Whitin algorithm are described.*
Key words: *Inventory control, optimization, Wagner-Whitin algorithm, logistics, manufacturing resource planning.*

Analiza osetljivosti Vagner-Vhitin algoritma. *Planiranje materijalnih potreba (MRP) igra važnu ulogu u poboljšanju efikasnosti proizvodnih kompanija. Na MRP rešenja sistema za planiranje resursa preduzeća (ERP) utiču i tehnološki i logistički parametri, ali korišćenjem dodatnih algoritama, kao što su Vagner-Vhitin ili Silver-Meal heuristika, moguće je uzeti u obzir više parametara. Ove heuristike mogu optimizovati rezultate MRP-a, posebno sa stanovišta troškova naručivanja, skladištenja i transporta. U okviru ovog članka opisan je uticaj parametara procesa na rezultat Vagner-Vhitin algoritma.*
Ključne reči: *Kontrola zaliha, optimizacija, Vagner-Vhitin algoritam, logistika, planiranje proizvodnih resursa.*

1. INTRODUCTION

Production logistics is a group of micro-logistics systems that is an integral part of enterprise logistics, and its operation is crucially influenced by related logistics systems such as purchasing, distribution and inverse logistics. There are strategies for all four functional areas of logistics. The integrated operation of these strategies is a prerequisite for the efficient operation of the production logistics system.

The logistics functions of production are usually defined from the raw material warehouse to the finished goods warehouse, and can be interpreted in terms of the raw materials, auxiliary materials, parts, assemblies, sub-assemblies, tools, equipment, technological and logistics resources, and human resources required to produce the finished product. The flow of materials and information follows all stages of the production process, including the necessary waiting and storage between each stage. Production logistics is responsible for providing the materials and technological resources needed to produce the finished product and for the coordinated management of the finished product production process to fulfil customer needs. In order to achieve this objective, production logistics has to perform a wide range of operations related to the material management: manufacturing, sourcing, inventory, packaging and inverse processes.

These logistics related operations of manufacturing systems are extensively influenced by the required materials, therefore the wide range of models and methods of material requirement planning influences not only the efficiency of the manufacturing operations but it has a great impact on the performance of the whole production system, including in-plant and external supply, material handling, logistics and human resource management. MRP can be described as an integration of three functional areas: inventory tracking, bill of materials (BOM) and master production scheduling. The efficiency can be increased through the integration of other functional areas, like machine capacity scheduling, forecasting of customers' demand or the while supply chain management, but in this case we are talking about MRP2 or ERP solutions. Other potential way to improve the solutions of MRP tools is the application of optimization algorithms to take more input parameters into consideration. The most known algorithms of increasing the efficiency of MRP solutions are the Silver-Meal heuristics and the Wagner-Whitin algorithm. The input parameters of these algorithms influences the supposed improvement of MRP solutions from cost efficiency point of view. This paper studies the sensitivity analysis of the Wagner-Whitin algorithm focusing on the cost efficincy. As the literature review section will show, the majority of the articles in the field of application of Wagner-Whitin algorithm are focusing on different application fields and only a few of them describe the impact of the input parameters on the objective function.

This paper is organized as follows. Section 2 presents a systematic literature review, which summarizes the research background of the Wagner-Whitin algorithm. Section 3 describes the model framework of Wagner-Whitin algorithm. Section 4 presents the results of the numerical analysis of two scenarios focusing on the impact of input parameters on the objective function. Conclusions and future research directions are discussed in Section 5.

2. LITERATURE REVIEW

2.1 Methodology

Within the frame of this chapter, the existing research results are analysed with a systematic literature review to find research gaps and main research directions. This systematic literature review includes three phases as follows: descriptive analysis of available research results and articles, content analysis, and consequences of the systematic literature review.

In this systematic literature review, the following main phases are performed: definition of the research questions, select sources from Scopus, reduce the number of articles by reading them and identifying the main topic if the number of identified articles is too high, analysing articles related to the research topic of this article, describing the main scientific results, and identifying research gaps.

Firstly, the relevant terms were defined. It is a crucial phase of the review because there is a wide range of excellent review articles in the field of production planning, scheduling and inventory control, and we didn't want to produce a similar review. It was not difficult to find the suitable keywords to find related articles, because the „TITLE (Wagner-Whitin)" keywords resulted 33 articles, which are focusing on the optimization potentials of Wagner-Whitin algorithm. It was not required to reduce the resulted number of articles, because 33 articles could be optimal for a research work. The search was conducted in April 2022; therefore, new articles may have been published regarding Wagner-Whitin algorithm since then.

2.2 Descriptive analysis

As Figure 1 demonstrates, the Wagner-Whitin algorithm-based optimization has been researched in the past 50 years. The first articles in this field were published in the 70s and 80s focusing on lot sizing problems, economic order quantity models (EOQ) and performance evaluation and comparison with Silver-Meal heuristics [23,24]. The number of published papers has been increased in the last years; it emphasizes the importance of this research field.

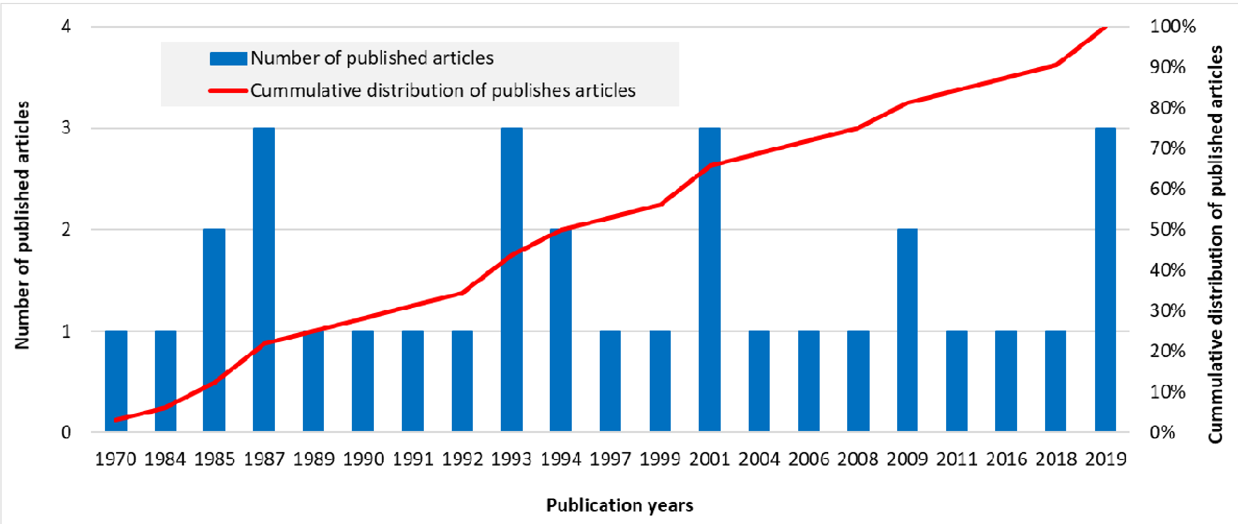

Fig. 1. Yearly distribution of articles focusing on Wagner-Whitin algorithm, based on the Scopus search TITLE (Wagner-Whitin)

The articles can be classified depending on the research area. Figure 2 shows the classification of these 33 articles considering nine subject areas of Scopus database. This classification shows that the majority of thee research works were performed in the field of engineering (manufacturing and production) while computer science and decision making focuses on the optimization aspects of these works. The importance of multidisciplinary sciences shows, that Wagner-Whitin algorithms-based lot sizing is important not only from performance of manufacturing systems, but also logistics, cost-efficiency, human resources and organizational point of view.

The published articles were analysed from the Scopus keywords point of view. The distribution of articles was described in the following categories: inventory control, lot sizing, algorithms, Wagner-Whitin algorithm, mathematical models, production control, optimization, constraints theory and cost analysis, cost accounting.

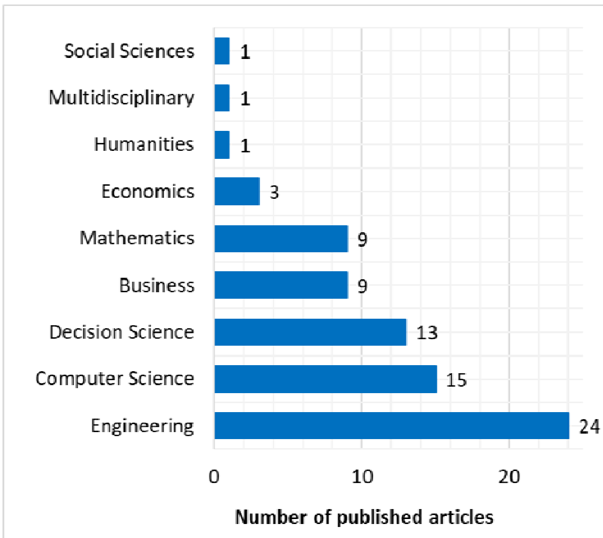

Fig. 2. Distribution of subject areas focusing on Wagner-Whitin algorithm, based on the Scopus search TITLE (Wagner-Whitin)

The distribution of the keywords is shown in Figure 3. As the categories show, the lot sizing problems are based on mathematical models and algorithms, where constraints can influence the complexity and suitable solution methods of lot sizing problems.

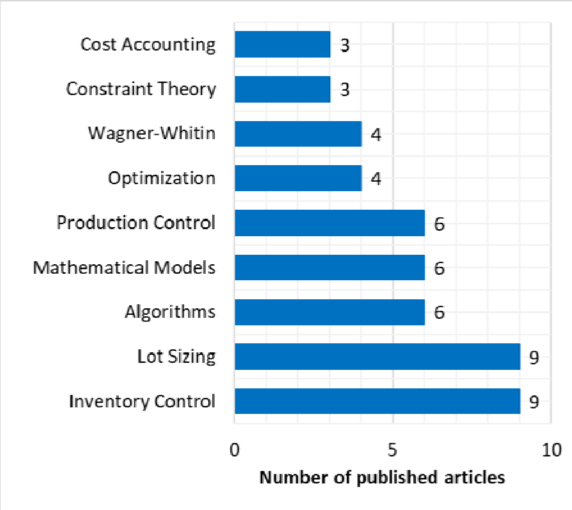

Fig. 3. Distribution of keywords focusing on Wagner-Whitin algorithm, based on the Scopus search TITLE (Wagner-Whitin)

2.3 Content analysis

The content analysis of the Wagner-Whitin algorithm related research works can be summarized as follows:

- A wide range of case studies shows the efficiency of Wagner-Whitin and Silver Meal algorithms, for example the optimal lot size at Diyala Public Company [3], stock control of natural resources [10], echelon stock formulation of arborescent distribution systems [11].
- The sensitivity analysis of Silver-Meal heuristics in the case of just-in-sequence supply related batches shows, that Silver-Meal heuristics can improve the performance of MRP, and it makes possible to decrease production cost with the rescheduling of production. The scenario analysis shows the impact of the specific warehousing cost on the total costs, set-up cost of production, warehousing cost, mean and deviation of preproduction weeks [25].
- The original Wagner-Whitin algorithm runs in $O(T^2)$ time [14,19], but researches show, that it is possible to develop a linear-time algorithm for Wagner-Whitin model, which runs in $O(T)$ time [18,22], depending on the number of time windows of the problem instance [4]. Computational results show, that Wagner-Whitin algorithm can solve dynamic lot sizing problems $O(T)$ time, but the Silver-Meal heuristics is about 15 times faster [23].
- The regeneration-point dynamic programming approach can be improved including minimum cost flow network formulation and standard forward dynamic-programming [1].
- In real manufacturing systems, the parameters (customers' demands, costs, capacities) are not deterministic. To solve the dynamic lot sizing problem in uncertain environment, it is possible to improve the Wagner-Whitin algorithm to consider stochastic costs with the information of probability density function of random costs using fuzzy models [2,6,8,13].
- Important application field of Wagner-Whitin algorithm is the lot sizing in the case of products with guarantee period [7].
- Research works show, that the algorithm is effective with and without backlogging [16], backorder [25], and the inventory holding and setup costs can be either fixed or dynamic [9].
- The economic order quantity models have convex objective functions, while dynamic lot sizing models are working with concave objective functions. Research work shows, that it is possible to reformulate the Wagner-Whitin algorithm to use convex objective function, where only the specific production or order costs are constant [21].
- The classical Wagner-Whitin model can be combined with the reverse Wagner-Whitin, where given returns of used products can be taken into consideration [12,15].
- The conventional Wagner-Whitin dynamic lot size model ignores reduction of lot size and total cost, while just-in-time philosophy strives for the goals of zero inventory at the end of each period and minimum total cost [17], so Wagner-Whitin algorithm can be used in the case of both conventional manufacturing system and just-in-time or just-in-sequence solutions [5].
- The efficiency of Wagner-Whitin algorithm can be improved by the integration of scheduling, product sequencing, and cycles of production formation in a group technology environment [20].

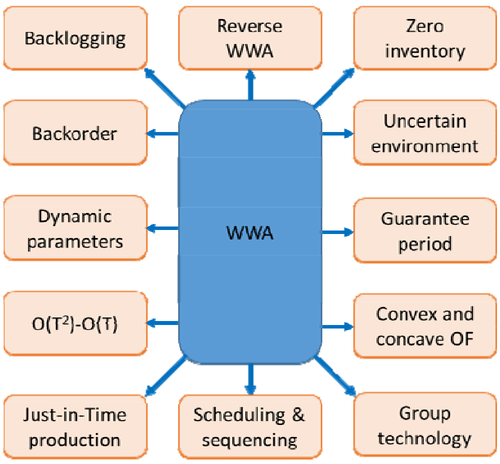

Fig. 4. The most important keywords of dynamic lot sizing problems using Wagner-Whitin algorithm

2.4 Conclusions of the literature review

The Wagner-Whitin algorithm represents an effective algorithm for dynamic lot sizing problems. The number of published articles was significantly increased and this result indicates the scientific potential of the Wagner-Whitin algorithm. The articles addressed different application of the algorithm and only a few of them describe the impact of input parameters from MRP on the performance of the method. Therefore, this research topic still needs more attention and research.

3. MATERIALS AND METHODS

The Wagner-Whitin algorithm gives an optimal solution for dynamic lot sizing problems. For the dynamic lot sizing problems solvable by the Wagner-Whitin algorithm, the following parameters are considered given:

- demand rate for each time windows: $\bar{f} = [f_i]$,
- setup cost for starting the production of one lot, which is a fixed cost linked to the production of a given type of product, independent of the quantity produced: k^{GYI},
- specific production cost: k^{GY},
- specific inventory holding or warehousing cost: k^T.

We take the following assumptions into consideration while using the Wagner-Whitin algorithm:

- There is no inventory available in the first time window, so the demand of this time window must be produced in this time window.
- There is no upper limit for the inventory in each time window, which means, that any large quantity of finished product can be stored in the warehouses and storages.
- The production capacity of each time windows is not constraints (no upper limit is given), therefore unlimited demand can be produced in each time windows.
- In a given time window, the amount of product produced must be exactly sufficient to meet the demand in each of the subsequent time window. The reason for this assumption lies in the fact that if a demand for time window α is only partially produced in an earlier time window β, then
  - on the one hand, we generate an additional storage cost that depends on the distance between the time window β of production and the time window α of consumption,
  - on the other hand, it is also necessary to produce in time window α, since the quantity needed in time window α is not available, which generates an additional setup cost for starting production.

If the demand for time window α is only partially produced in an earlier time window β, then the additional cost for the quantity produced can be defined as follows: $$C_+ = k^T(\alpha - \beta)\rho \qquad (1)$$

The algorithm is based on a recursive method that can be used to determine the optimal total cost during any arbitrary time window i and all other time windows thereafter: $$K_i = \min_{p=0 \dots i_{max}-i}(C_{ip} + K_{i+p+1}) \qquad (2)$$

where K_i is the total cost between time window i. and time window i_{max}, i_{max} is the upper limit of predefined time windows, where the exact production demand is given, C_{ip} is the total cost between time windows i. and time windows $i + p$.

We can compute the value of the total cost between time windows i. and time windows $i + p$ as follows:

$$C_{ip} = k^{GYI} + k^{GY} \sum_{\delta=0}^{p} f_{i+\delta} + k^T \sum_{\delta=1}^{p} \delta f_{i+\delta} \qquad (3)$$

The recursive algorithm works on the simple principle of first determining the total costs for the last time window using equation (1):

$$K_{i_{max}} = C_{i_{max}0} + K_{i_{max}+1} \qquad (4)$$

where $K_{i_{max}+1} = 0$ and $C_{i_{max}0} = k^{GYI} + k^{GY} f_{i_{max}}$, because if production takes place in the last time window, only the quantity for that time window can be produced, so there is no storage cost.

Then, applying equation (1) backwards over the time windows, the optimal value of K_i can be determined, which also determines the optimal production batch size for time window i and all subsequent time windows. Once the value of K_1 is determined, the solution to the overall dynamic batch size problem is given by determining the optimal production batch size for all time windows. It is important to note that to determine the optimal production schedule, the values of K_i must be used such that

- in the first step, based on the value of K_1, the number of subsequent time windows must be determined, for them the required demand is produced in the first time window, and then, assuming that this value is ε_1, we can use the value of K_{ε_1+1} to make the next optimal decision, i.e., how many additional demand in the subsequent time windows we should satisfy in the time window $\varepsilon_1 + 1$,
- in the next steps, based on the value of K_{ε_1+1}, the number of subsequent time windows must be determined; the required demand is produced in the time window $\varepsilon_1 + 1$; assuming that this value is ε_2, we can use the value of K_{ε_2+1} to make the next optimal decision, i.e., how many additional demand in the subsequent time windows we should satisfy in the time window $\varepsilon_2 + 1$. The procedure is continued until the optimal lot size is determined for each time window.

4. RESULTS

Within the frame of this chapter, the results of the sensitivity analysis of the Wagner-Whitin algorithm are described. The first scenario demonstrates the optimization results of Wagner-Whitin algorithm, where the setup cost for production is 250 EUR, the specific inventory holding cost is 2 EUR and the input parameters for demands resulted by material requirement planning are shown in Figure 5.

Week	1	2	3	4	5
Demand	120	240	320	52	250
Week	6	7	8	9	10
Demand	47	85	122	75	60

Fig. 5. Production demands for Scenario 1 lot sizing problem.

In the case of material requirement planning no inventory holding cost can be defined, because the demands are produced just-in-time, therefore the total cost of the material requirement is the sum of setup costs for production, which is 2500 EUR. Using the Wagner-Whitin algorithm, we can find a more cost-efficient solution in order to fulfil customers' demands, where through merging more production setup into one production setup the total setup cost of production operations can be decreased, while the inventory holding cost will be increased, because in this case we are not producing just-in-time. The numerical results of this first scenario depicted in Figure 6 shows, that just-in-time or just-in-sequence solutions are not always the most cost efficient solutions for material requirement problems.

Time window	Demand span	Inventory holding cost	Predecessor cost	Total cost
10	10	0	0	250
9	9	0	250	500
9	9-10	120	0	370
8	8	0	370	620
8	8-9	150	250	650
8	8-10	390	0	640
7	7	0	620	870
7	7-8	244	370	864
7	7-9	544	250	1044
7	7-10	904	0	1154
6	6	0	864	1114
6	6-7	170	620	1040
6	6-8	658	370	1278
6	6-9	1108	250	1608
6	6-10	1588	0	1838
5	5	0	1040	1290
5	5-6	94	864	1208
5	5-7	434	620	1304
5	5-8	1166	370	1786
5	5-9	1766	250	2266
5	5-10	2366	0	2616
4	4	0	2156	2822
4	4-5	500	1892	3058
4	4-6	688	1570	2924
4	4-7	1198	1056	2920
4	4-8	2174	786	3626
4	4-9	2924	666	4256
4	4-10	3644	0	4310
3	3	0	2822	3488
3	3-4	104	2156	2926
3	3-5	1104	1892	3662
3	3-6	1386	1570	3622
3	3-7	2066	1056	3788
3	3-8	3286	786	4738
3	3-9	4186	666	5518
3	3-10	5026	0	5692
2	2	0	2926	3592
2	2-3	640	2822	4128
2	2-4	848	2156	3670
2	2-5	1348	1892	3906
2	2-6	1724	1570	3960
2	2-7	2574	1056	4296
2	2-8	4038	786	5490
2	2-9	5088	666	6420
2	2-10	6048	0	6714
1	1	0	3592	4258
1	1-2	480	2926	4072
1	1-3	1760	2822	5248
1	1-4	2072	2156	4894
1	1-5	4072	1892	6630
1	1-6	4542	1570	6778
1	1-7	5562	1056	7284
1	1-8	7270	786	8722
1	1-9	8470	666	9802
1	1-10	9550	0	10216

Fig. 6. Results of the optimization of lot sizing problem in Scenario 1 using Wagner-Whitin algorithm.

The second scenario analyses the impact of input parameters of Wagner-Whitin algorithm on the cost functions. As Figure 7 shows, the increased setup cost of production increases the total cost, but the function is non-linear. The specific inventory cost also increases the total cost in a non-linear way, because using the Wagner-Whitin algorithm the changes in one type of costs influences all types of costs in the case of the optimal solution of the dynamic lot sizing problem.

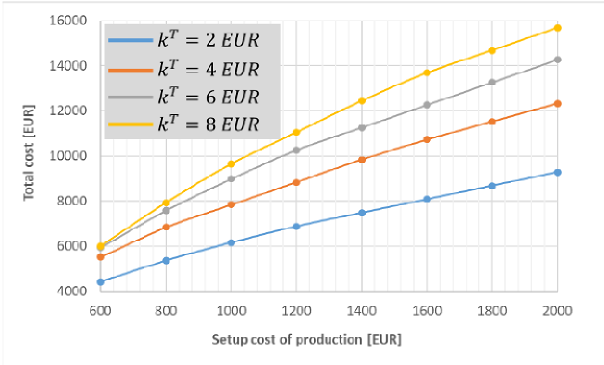

Fig. 7. Impact of setup cost of production and specific inventory holding cost on the total cost of the optimal solution using Wagner-Whitin algorithm in Scenario 2.

Figure 8 demonstrates the impact of the distribution of customers' demands on the total cost and its percentual increase. In the case of lower specific inventory holding cost, the increased deviation of the normal distribution function of customers' demands lead to increased total costs, while in the case of higher specific inventory holding cost, the increased deviation leads to decreased total cost. This short analysis of the most important input parameters of Wagner-Whitin algorithm shows, that the improved solution of dynamic lot sizing problem is influenced by a non-linear way from input parameters regarding specific costs and demands.

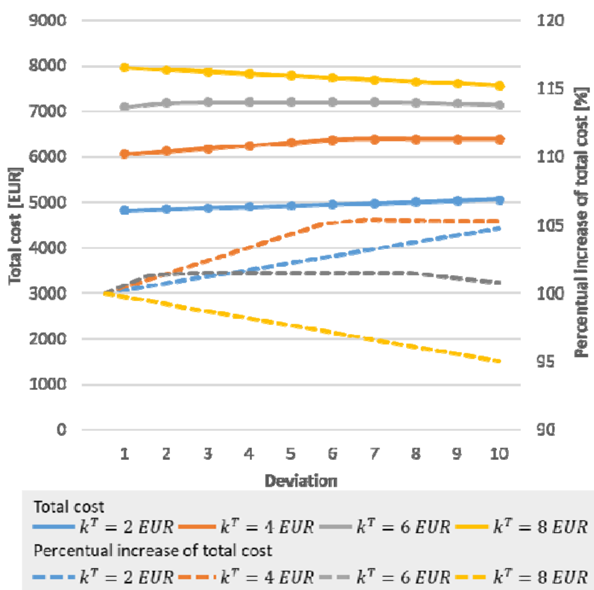

Fig. 8. Impact of the deviation of normal distribution-based demands of customers on the total cost and its percentual increase/decrease using Wagner-Whitin algorithm in Scenario 2.

5. CONCLUSIONS

The Wagner-Whitin algorithm is a suitable tool to improve the results of material requirement planning, because setup of production, setup of order and inventory holding costs can be taken into consideration. As the sensitivity analysis shows, the results of the optimization and the value of the objective function is influenced by the parameters of the dynamic lot sizing problem, including mean and deviance of customers' demands, setup and inventory holding cost.

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Author: **Associate professor Tamás Bányai PhD,** University of Miskolc, Faculty of Mechanical Engineering and Informatics, Institute of Logistics, 3515 Miskolc, Hungary, Phone.: +36 46 565-111.
E-mail: alttamas@uni-miskolc.hu