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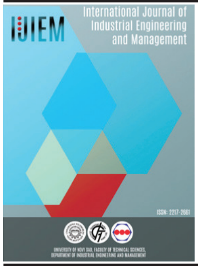

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Original research article

Innovative Quality Determination by Integrating Process Incapability Index into Variable Sampling Plan Design

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ABSTRACT

This study presents an innovative acceptance sampling plan for variables that integrates the process incapability index with a resubmitted sampling scheme (C_{pp}-RSP) to enhance the efficiency of lot sentencing. While conventional single-sampling plans (SSP) are attractive for their simplicity, they often require large fixed sample sizes, which generate substantial inspection costs for both producers and customers. To overcome this constraint, the proposed sampling plan leverages the statistical properties of C_{pp} to construct a nonlinear optimization model that determines the optimal plan parameters subject to prescribed quality standards and risks for producers and consumers. This parametric framework yields closed-form operating characteristic (OC) and average sample number (ASN) functions, leading to practical design tables for different quality and risk settings. Numerical investigations demonstrate that across a broad range of quality levels and risk combinations, the C_{pp}-RSP reduces the per-stage sample size required by roughly 25–50% compared with the corresponding SSP. These results indicate that incorporating capability indices and resubmission mechanisms into acceptance sampling can substantially reduce routine inspection effort while maintaining rigorous control of decision risks, offering a promising direction for more adaptive and economical quality control systems.

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1. Introduction

The growth of industrial production systems has revealed substantial advancements and innovations resulting from the digital transformation of the Fourth Industrial Revolution. The technical improvements in this arena have reduced costs for producers, increased scalability, maximized operating efficiency, improved regulatory compliance, and boosted product quality [1]. Quality management systems, as an essential element of these technical breakthroughs, are vital in guaranteeing product quality before customer delivery [2], [3]. Statistical Quality Control (SQC) is a crucial tool for evaluating and regulating production lines, therefore significantly enhancing quality management systems and maintaining the quality of processes and products. The SQC system comprises

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two primary components: (1) control charts for real-time assessment and oversight of production processes, and (2) acceptance sampling plans to guarantee the quality of raw materials, semi-finished products, and final goods before delivery to customers [4]. The incorporation of acceptance sampling plans into the SQC acts as a conciliatory approach, reconciling two divergent inspection techniques (100% inspection and zero inspection) often utilized across various industries. These methodologies aim to assess compliance with specified requirements by evaluating critical quality characteristics of products. In addition, over the last three years, a comprehensive study has been undertaken on the development of acceptance sampling strategies from several viewpoints, including [5]-[14]. A thorough examination of acceptance sampling plans along with process capability indices from the preceding decade is presented in the review study by Yum [15].

Sampling plans were initially formulated considering only a single stage, commonly referred to as a Single Sampling Plan (SSP). The utilization of SSPs is favored across various industries due to their simplicity and ease of implementation[4]. Despite their popularity and ease of use, SSPs possess several drawbacks, notably the requirement for large sample sizes, which consequently leads to increased inspection costs. Furthermore, due to their single-stage nature, SSPs place a burden on both producers and consumers, as there is no alternative means to evaluate lot quality [16], [17]. Occasionally, this psychological issue prompts producers to question the validity of single-sample results, and resampling may be conducted in accordance with the provisions stipulated in the agreements.

To address the above issues, Govindaraju and Ganesalingam [18] proposed a resubmitted sampling plan strategy that allows for the possibility of resubmitting a lot if it is rejected based on the initial sampling method. It is assumed that rehandling and reprocessing do not enhance product quality during resubmission. In recent years, extensive research has been conducted on the formulation of resubmitted sampling plans from various perspectives, with a significant focus on capability indices, including contributions from researchers [14], [19]-[27]. Wu et al. [22] developed a resampling strategy utilizing advanced capability indices. Kurniati et al. [23] developed a resubmitted sampling plan based on a one-sided capability index perspective. Using a linear profile approach, Wang [12] established a resampling plan under a resubmitted model. Seifi and Nezhad [13] employed a Bayesian approach in developing a resampling plan with resubmission. Recent research on resubmitted models was conducted by Yen et al. [27], integrating it with the coefficient of variation. Additionally, most studies on resubmitted sampling plans evaluate product quality solely based on process outcomes, typically assessing performance by indicating the percentage of products conforming to specifications, as developed by Wu et al. [19]; however, they do not differentiate between products within specification limits. Thus, Wu et al. [21] elaborated the resubmitted sampling plan based on C_{pm}. Lastly, Darmawan et al. [14] developed a resubmission model of a sampling plan based on the process loss index. Therefore, we proposed an innovative quality sentencing approach for variable sampling plans, integrating the process incapability index. The incapability index exhibits enhanced statistical properties relative to earlier indices (e.g., C_{pm}), since it eliminates the necessity for reciprocal translation of process mean and variance, and provides a significant differentiation between data about process accuracy and process precision. Besides that, to the best of our knowledge, the exploration of developing resubmitted sampling plans based on process incapability indices remains underexplored.

The ensuing sections of this article are structured in the following sequence. Section 2 elucidates the literature review of the incapability index and an examination of the statistical properties of its estimator. Furthermore, Section 3 provides a comprehensive explanation of the operational mechanics of the proposed system, including the OC and ASN functions, along with a mathematical framework for determining plan parameters. Section 4 offers an extensive study and discourse on plan parameters across many scenarios, followed by a comparative evaluation of the developed sample scheme. In conclusion, Section 5 summarizes the principal findings and conclusions of this study.

2. Literature Review of Process Incapability Index C_{pp}

The quadratic loss function index, C_{pm}, was developed to assess and monitor production floor performance with the quality loss function proposed by Hsiang and Taguchi [28]. Nevertheless, owing to the analytical intractability of the statistical features of the C_{pm} index [29], [30], Greenwich and Jahr-Schaffrath [31] popularized the process incapability index, C_{pp}, for assessing production process performance. Conceptually, C_{pp} measures the overall degree to which a process fails to meet its specification requirements, and it decomposes additively into an *inaccuracy* component, capturing the deviation of the process mean from the target, and an *imprecision* component, capturing excessive dispersion around the mean. Lower C_{pp} values, therefore, correspond to more capable processes (i.e., lower incapability), whereas higher values indicate poorer process performance. For ease of interpretation, C_{pp} values close to zero correspond to highly capable ('good') processes, whereas values approaching or exceeding unity represent processes with substantial incapability ('poor' processes).

The C_{pp} index is defined as $C_{pp} = C_{ia} + C_{ip}$, where C_{ia} (inaccuracy index) and C_{ip} (imprecision index) measure the levels of process accuracy and precision, respectively. Consequently, the C_{pp} index may be expressed as:

$$C_{pp} = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2 \quad (1)$$

where μ represents the process mean, σ denotes the process standard deviation, T signifies the target value, D is defined as $d/3$, with d being $(USL-LSL)/2$, the half specification width, and USL and LSL refer to the upper and lower specification limits, respectively. Additionally, C_{ia} is calculated as $(\mathrm{M}-T)/D^2$, where M is the middle point of the specification interval $(USL+LSL)/2$, and C_{ip} is expressed as σ^2/D^2. The C_{pp} index, as a simple adaptation of the C_{pm} index, provides a discrete separation of data pertaining to the accuracy and precision of the process. The mathematical connections $C_{pp} = (C_{pm})^{-2}$, $C_{ia} = (3-3C_a)2$, $(C_a = 1-|\mu - T|/d)$, and $C_{ip} = (C_p)^{-2}$ can be obtained using the definition described above. The advantage of C_{pp} over C_{pm} is that, because it does not need a reciprocal translation of process mean and variance, its estimator has better statistical properties than C_{pm}'s. Only a lower bound evaluation of the process yield, *yield* $\geq 2\Phi$ (*3* $\times$ *index value*)-1, may be provided by the capability index C_{pk} and C_{pm}. It is crucial to remember that a process's maximum potential yield is unimportant. The process yield would be equal to or greater than $2\Phi(3C)-1$ if the index value were C. Index C_{pp} can be used as a lower bound estimate for the process yield (yield > $2\Phi(3(1/C_{pp})^{1/2})-1$), as the index C_{pp} can be expressed as $(C_{pm})^{-2}$. Table 1 presents a compilation of frequently utilized C_{pp} values alongside their matching C_{pm} values.

In reality, population parameters are often unknown; therefore, a data sample is collected to estimate these parameters for the C_{pp} index. To evaluate process capability utilizing the incapability index C_{pp}, a natural estimator, $\hat{C}_{pp}$, is established, with the maximum likelihood estimation (MLE) of C_{pp} articulated as:

$$\hat{C}_{PP} = \left(\frac{(\bar{X}-T)^2}{D^2}\right) + \frac{S_n^2}{D^2} = \left(\frac{(\bar{X}-T)^2}{D^2}\right) + \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i-\bar{X})^2}{D^2} = \frac{\sum_{i=1}^{n}(X_i-T)^2}{nD^2}. \quad (2)$$

where $\bar{X} = \sum_{i=1}^{n} X_i / n$, $S_n^2 = \sum_{i=1}^{n}(X_i-\bar{X})^2 / n$.

According to Equation 2 and the premise of normally distributed data, the mathematical correlation between the C_{pp} index and its estimator may be expressed as follows:

$$\frac{\hat{C}_{PP}}{C_{PP}} = \frac{\sum_{i=1}^{n}(X_i-T)^2}{nD^2} \times \frac{D^2}{\left[\sigma + (\mu-T)^2\right]} = \frac{\dfrac{\sum_{i=1}^{n}(X_i-T)^2}{\sigma^2}}{n\left[1+\left(\dfrac{(\mu-T)^2}{\sigma^2}\right)\right]} = \frac{\chi^2_{n,\delta}}{n+\delta} \quad (3)$$

where $\delta = n\xi^2 = n(\mu\text{-}T)^2/\sigma^2$. Consequently, $\hat{C}_{pp}$ can be allocated as $C_{pp}\chi^2_{n,\delta} / n+\delta$. It is worth expressing that when the value of (ξ) equals zero, it signifies

Table 1. Frequently used levels of C_{pp} and their respective C_{pm} levels

Condition	C_{pm}	C_{pp}
Incapable	0.50	4
Capable	1.00	1
Satisfactory	1.33	0.5653
Good	1.50	0.4444
Excellent	1.67	0.3586
Super	2.00	0.25

the process mean is precisely at the target value (T). Consequently, the cumulative distribution function (CDF) of $\hat{C}_{pp}$ can be established as

$$F_{C_{pp}}(y) = P(\hat{C}_{pp} \le y) = P\left(\chi^2_{n,n(1+\xi^2)} \le \frac{n(1+\xi^2)y}{C_{pp}} \right). \quad (4)$$

Furthermore, some research [32]-[48] that focused on the development of the process incapability index since its introduction.

3. Methodology

3.1 The Flowchart Sampling Plan C_{pp}-RSP

This section outlines the development of a resubmitted sample strategy grounded in the process incapability index. This research assumes that the quality characteristic of interest follows a normal distribution and possesses dual specification limits. When the quality characteristic surpasses the established standard limits, the product is deemed nonconforming. The existing quality improvement strategy is based on the premise of minimizing the rate of nonconformance and divergence from the target value. In addressing the previously described issue, the formulation of the acceptance sampling plan incorporates the process incapability index (C_{pp}), which serves as an appropriate quality standard. Figure 1 illustrates the sequential operational mechanism of the proposed strategy using a flowchart, which is detailed below:

1. Define the necessary characteristics as stipulated in the contract, including the producer's risk α, consumer's risk β, acceptable quality level (AQL) and rejectable quality level (RQL) represented by the C_{pp} index (denoted as c_{AQL} and c_{RQL}, respectively), along with the number of resampling instances (m). In addition, define the final non-acceptance statement that a lot that is finally rejected is handled in accordance with the contract (e.g., returned to the supplier, 100% screening with rework/repair, downgraded or price-conceded, or scrap).
2. Acquire the plan parameters sample size (n) and critical value of acceptance (c_0), by examining the tables according to the designated criteria.
3. Conduct the first sampling and examine a random sample of size (n) from the lot to compute $\hat{C}_{pp}$.
4. Decide about the submitted lot in accordance with the regulation: Accept the lot if $\hat{C}_{pp} < c_0$; (ii) If not (non-acceptance condition), establish $s = 1$ and proceed to step 5.
5. Conduct the subsequent inspection, examine a random sample size (n), and calculate $\hat{C}_{pp}$.
6. Verify the sampling times (s). After the final non-acceptance (i.e., rejection after s is more than or equal to m-1 allowed resubmissions), the lot is finally rejected and disposed of according to the agreed-upon quality contract procedure. If not, go to step 7.

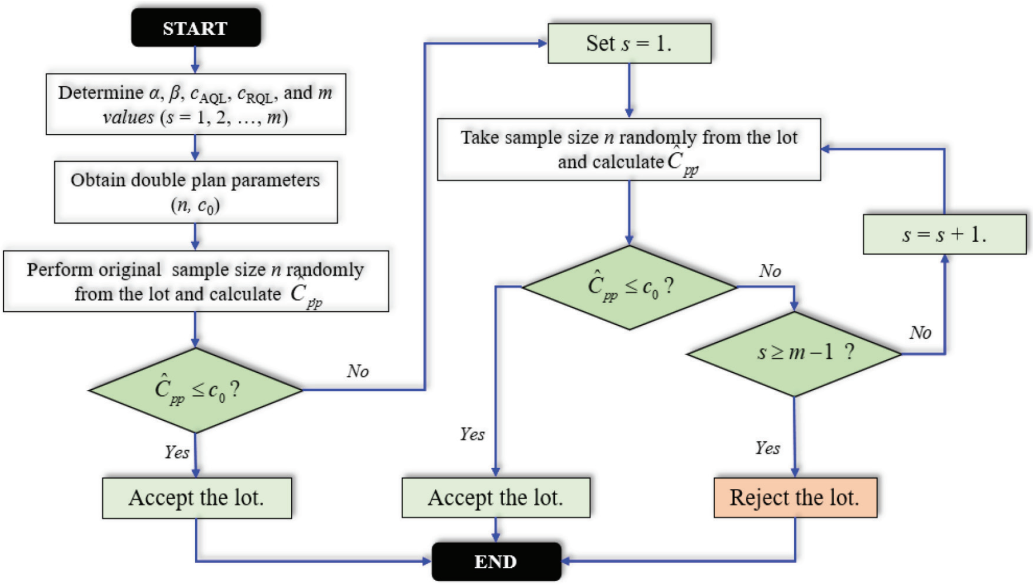


Figure 1. Flowchart for the operational method of C_{pp}-RSP

7. Decide about the submitted lot following the established rule: (i) Accept the lot if $\hat{C}_{pp} \leq c_0$; (ii) Otherwise, revert to Step 5 and proceed accordingly.

3.2 Nonlinear Optimization Model

Probability functions can be derived from the sample distribution of $\hat{C}_{pp}$, as outlined below. When the lot's quality is specified as $C_{pp} = c$, the probability of lot acceptance at the sampling time (s=*1*, *2*, …, m) may be computed as:

$$P_a(c) = P(\hat{C}_{PP} \leq c_0 \left| C_{PP} = c) = 1 - P(\hat{C}_{PP} > c_0 \right\| C_{PP} = c) =$$

$$= P\left(\chi^2_{n,n\left(1+\xi^2\right)} \leq \frac{n\left(1+\xi^2\right)c_0}{c}\right). \quad (5)$$

According to the operational methods, the functional properties of the proposed system C_{pp}-RSP, $\pi_A(c)$, can be articulated as follows:

$$\pi_A(c) = P_a(c) + [1-P_a(c)]P_a(c) +$$
$$+[1-P_a(c)]^2 P_a(c) + \ldots + [1-P_a(c)]^{m-1} P_a(c)$$
$$= \frac{P_a(c)(1-[1-P_a(c)]^m)}{1-[1-P_a(c)]} = 1-[1-P_a(c)]^m \quad (6)$$

It is noteworthy that the C_{pp}-RSP will diminish to a variable SSP contingent upon C_{pp} (i.e., C_{pp}-RSP) under the condition m=1 [42].

In the proposed C_{pp}-RSP, resampling for each submitted lot is restricted to a maximum of $m - 1$ sampling instances until the final judgment is made. Consequently, when evaluating the efficacy of the sampling strategy, it is more logical and prudent to consider the ASN. An ASN indicates the anticipated or average number of sample units necessary for inspection to reach a definitive conclusion on the lot's disposition. The formulation of the ASN for the proposed C_{pp}-RSP under C_{pp}=c may be determined as follows.

$$\text{ASN}(c) = n + n[1-P_a(c)] + n[1-P_a(c)]^2 + \ldots + n[1-P_a(c)]^{m-1}$$
$$= \frac{n(1-[1-P_a(c)]^m)}{1-[1-P_a(c)]} = \frac{n(1-[1-P_a(c)]^m)}{P_a(c)}. \quad (7)$$

As in the prior case, a carefully crafted strategy must satisfy the dual criteria on the OC curve: (i) the lot must be ultimately accepted at its quality level with a minimum probability of 100(1 – α)% after m – 1 resubmissions; (ii) the lot must be ultimately accepted at its quality level with a probability surpassing 100β%. Therefore, the pair parameters (n, c_0) of the resubmitted sample plan, which incorporates the process incapability index, must fulfill the following two criteria:

$$\pi_A(c_{\text{AQL}}) = 1-[1-P_a(c_{\text{AQL}})]^m \geq 1-\alpha, \quad (8)$$

and

$$\pi_A(c_{\text{RQL}}) = 1-[1-P_a(c_{\text{RQL}})]^m \leq \beta, \quad (9)$$

$$c_0 > 0, \ n \geq 2.$$

Consequently, the necessary sample size (n) and critical acceptance (c_0) for inspection can be ascertained by concurrently solving the following two nonlinear equations:

$$f_1(n,c_0) = [1-P_a(c_{\text{AQL}})]^m - \alpha \leq 0 \quad (10)$$

and

$$f_2(n,c_0) = [1-P_a(c_{\text{RQL}})]^m - (1-\beta) \geq 0 \quad (11)$$

As previously stated, the (ξ) value in the CDF of $\hat{C}_{pp}$, as articulated in Eq. (4), equals zero, signifying that the process mean corresponds with the process target (T). Consequently, the plan parameters are obtained in this study by solving the previously described nonlinear mathematical model, utilizing the suggested configurations

4. Results and Discussion

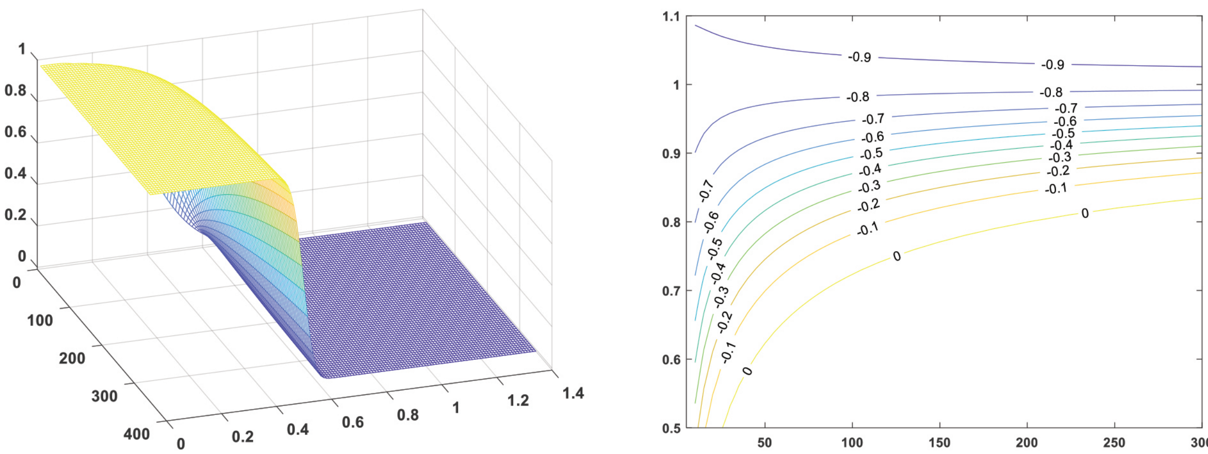
In conducting simulations for analysis and discussion, we use the *'fsolve'* application integrated into the MATLAB software to simulate the sensitivity analysis and obtain the values of the parameters (n, c_0) from Equations 10 and 11. To produce data in tabular format, the determination of parameters (n, c_0) is based on a specific combination of quality levels and risk levels, allowing them to be used and utilized in the real world.

4.1 Determination of Plan parameters (n, c_0)

Equations 10 and 11 are utilized as non-linear mathematical representations to visually demonstrate the determination of optimal parameters (n, c_0). To visually ascertain parameters, it is necessary to identify specific quality and risk levels, such as quality level values (c_{AQL}, c_{RQL}) = (0.5917, 1.0), and risk levels (α, β) = (0.01, 0.05), with m set to 3. Subsequently, these combinations facilitate the visual generation of stages for the acquisition of optimal parameters (n, c_0). Figure 2 presents the visualisation of (a) the surface plot and (b) the contour plot for equation 10. Figure 3 illustrates (a) the surface plot and (b) the contour plot for equation 11. The meeting to ascertain the optimal point is represented as visually integrating equations 10 and 11, as illustrated in Figure 4: (a) surface plot and (b) contour plot.

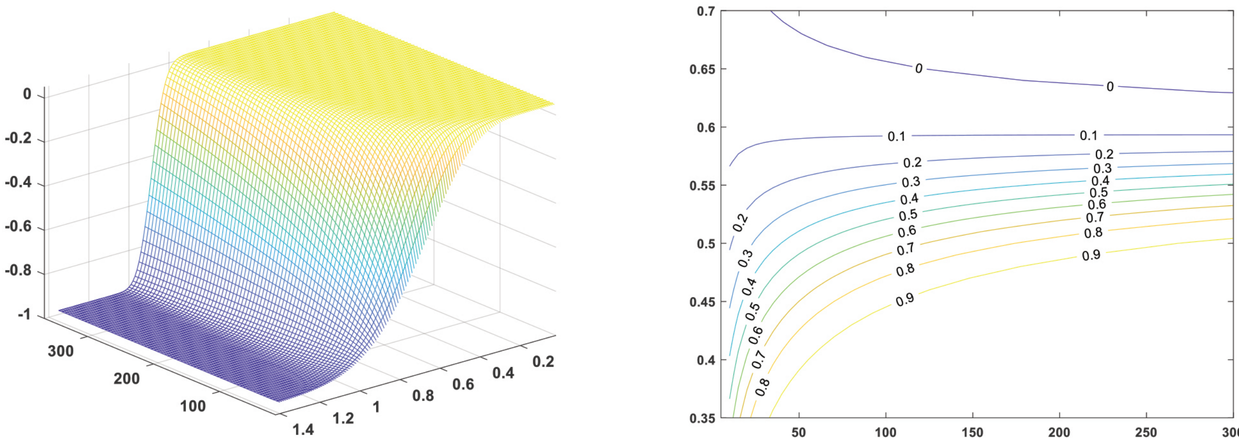
The vertex point parameter is identified as $(n, c_0) = (68, 0.6695)$ for the specified combination of quality and risk previously discussed. The vertex point at $(n, c_0) = (68, 0.6695)$ indicates the minimum sample size within the efficient frontier solution, thereby establishing it as the most effective solution for this scenario in terms of economic value and time efficiency. A simulation is conducted and subsequently summarised in a table for the parameter values (n, c_0) listed in Tables 2, 3, and 4, respectively, incorporating various combinations of quality levels and risk levels with setting values of $m = 2$, 3, and 4. The arrangement of these tables enables quality engineers or practitioners to determine parameter values in alignment with the agreement established between the producer and the consumer [41].

For instance, the m value is 2. The quality levels (c_{AQL}, c_{RQL}) are specified as (0.5917, 1.00), and the risk levels (α, β) are selected as (0.01, 0.05). Consequently, Table 2 demonstrates that = (80, 0.7146) for $m = 2$. Therefore, a random sample of 80 items from the submitted lot should be selected, and the C_{pp} should be calculated. The lot will be accepted if the C_{pp} is less than or equal to 0.7146. Otherwise, the lot will be rejected if the condition is also non-accep-



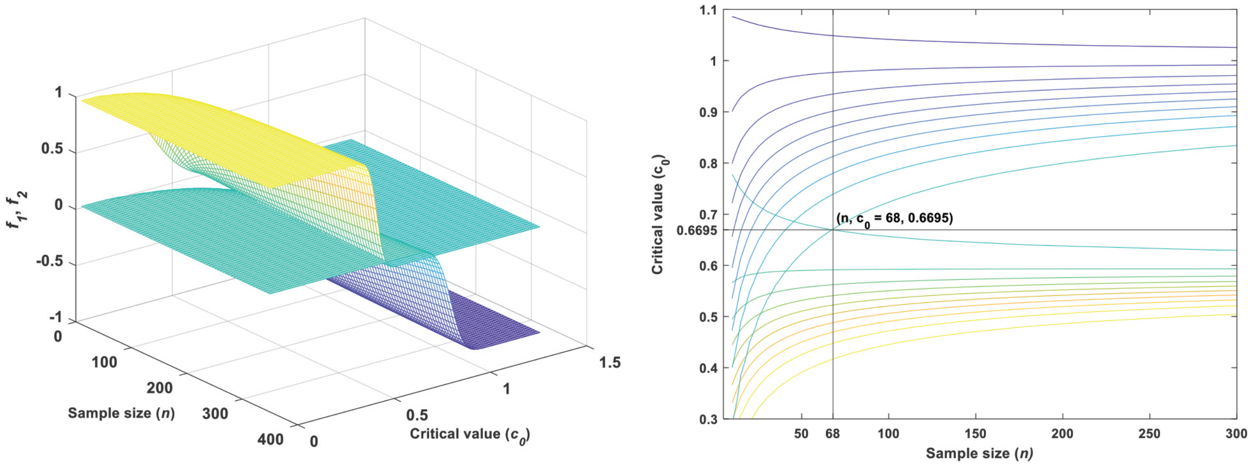
(a) Surface plot (b) Contour plot

Figure 2. (a) The surface plot and (b) Contour plot of $f_1(n, c_0)$ for $m = 3$ under $(c_{AQL}, c_{RQL}) = (0.5917, 1.0)$, and $(\alpha, \beta) = (0.01, 0.05)$



(a) Surface plot (b) Contour plot

Figure 3. (a) The surface plot and (b) Contour plot of $f_2(n, c_0)$ for $m = 3$ under $(c_{AQL}, c_{RQL}) = (0.5917, 1.0)$, and $(\alpha, \beta) = (0.01, 0.05)$



(a) Surface plot (b) Contour plot

Figure 4. (a) The surface plot and (b) contour plot of two nonlinear formulations for $m = 3$ under $(c_{AQL}, c_{RQL}) = (0.5917, 1.0)$, and $(\alpha, \beta) = (0.01, 0.05)$

tance (i.e., C_{pp}>0.7146), and a single resubmission is permitted. Alternatively, when the practitioner establishes $m = 3$, the pair parameters can be determined as $(n, c_0) = (68, 0.6695)$. This indicates that the customer authorizes resubmission if the initial inspection is not approved [19, 20, 21]. A complete lot will be denied if the initial two resubmissions are not accepted with a sample size of 68.

The three tables present the plan parameters for the C_{pp}-RSP under three different sampling times of inspection, $m = 2$ (Table 2), $m = 3$ (Table 3), and $m = 4$ (Table 4). Each table organizes results by producer's risk (α) and consumer's risk (β) in rows, and by pairs of acceptable quality level (c_{AQL}) and rejectable quality level (c_{RQL}) in columns. Within each (c_{AQL}, c_{RQL}) block, the sampling plan is specified by the sample size n and the acceptance threshold c_0 that jointly satisfy $P_a(AQL) = 1 - \alpha$ and $P_a(RQL) \leq \beta$.

Increasing the number of sampling times for inspection from two to four systematically reduces the required sample size (n) and slightly relaxes the acceptance threshold (c_0). This effect arises because an additional sampling time allows for risk to be spread over more decisions, improving overall inspection efficiency while still meeting the prescribed operating characteristic requirements [23]. For instance, at $\alpha = 0.05$ and $\beta = 0.05$ with $(c_{AQL}, c_{RQL}) = (0.4444, 0.5917)$, the double sampling times require $n = 189$ and $c_0 = 0.4785$; the triple sampling times reduce the sample to $n = 159$ and $c_0 = 0.4595$.

In addition, the tables demonstrate that a decrease in the risks taken by the producer and/or the customer correlates with an increase in the required sample size. This phenomenon can be explained by the necessity of a larger sample size to assess the lots, as this reduces the probability of misclassifying good lots as bad and vice versa [25], [27]. The associated critical value (c_0) exhibits a positive correlation with increasing β-risk and a negative correlation with rising α-risk, assuming all other factors remain constant.

Table 2. Plan parameters of C_{pp}-RSP under $m = 2$

(c_{AQL}, c_{RQL})		(0.5917, 1.0)		(0.4444, 0.5917)		(0.3673, 0.4444)		(0.2500, 0.3673)	
α	β	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	116	0.6935	376	0.4864	838	0.3905	211	0.2817
	0.05	80	0.7146	262	0.4949	586	0.3951	147	0.2881
	0.10	64	0.7294	211	0.5008	473	0.3983	118	0.2926
0.05	0.01	90	0.6568	287	0.4722	635	0.3828	162	0.2706
	0.05	59	0.6716	189	0.4785	419	0.3863	106	0.2754
	0.10	45	0.6821	146	0.4831	324	0.3889	82	0.2787
0.10	0.01	77	0.6332	244	0.4627	537	0.3777	138	0.2634
	0.05	49	0.6426	154	0.4671	340	0.3802	87	0.2666
	0.10	37	0.6492	116	0.4703	255	0.3821	66	0.2689

Table 3. Plan parameters of C_{pp}-RSP under $m = 3$

(c_{AQL}, c_{RQL})		(0.5917, 1.0)		(0.4444, 0.5917)		(0.3673, 0.4444)		(0.2500, 0.3673)	
α	β	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	99	0.6564	316	0.4719	700	0.3826	179	0.2705
	0.05	68	0.6695	217	0.4775	481	0.3858	122	0.2746
	0.10	54	0.6784	173	0.4813	385	0.3879	98	0.2775
0.05	0.01	78	0.6190	245	0.4568	539	0.3744	140	0.2590
	0.05	51	0.6241	159	0.4595	349	0.3760	90	0.2608
	0.10	39	0.6275	122	0.4613	268	0.3771	69	0.2621
0.10	0.01	68	0.5950	210	0.4469	460	0.3690	120	0.2515
	0.05	42	0.5939	131	0.4471	286	0.3692	75	0.2515
	0.10	32	0.5925	98	0.4471	213	0.3694	56	0.2513

Table 4. Plan parameters of C_{pp}-RSP under $m = 4$

(c_{AQL}, c_{RQL})		(0.5917, 1.0)		(0.4444, 0.5917)		(0.3673, 0.4444)		(0.2500, 0.3673)	
α	β	n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	90	0.6305	282	0.4615	622	0.3770	160	0.2625
	0.05	61	0.6379	192	0.4649	423	0.3789	109	0.2650
	0.10	48	0.6428	153	0.4672	337	0.3803	87	0.2666
0.05	0.01	71	0.5929	221	0.4459	484	0.3684	127	0.2508
	0.05	46	0.5915	142	0.4459	311	0.3685	82	0.2506
	0.10	35	0.5900	109	0.4458	237	0.3686	62	0.2504
0.10	0.01	62	0.5689	191	0.4358	415	0.3627	110	0.2432
	0.05	39	0.5608	118	0.4329	256	0.3613	68	0.2409
	0.10	29	0.5541	88	0.4306	190	0.3601	51	0.2390

The width of the interval between c_{AQL} and c_{RQL} also strongly influences both n and c_0. Wider intervals (e.g., (0.5917, 1.0000)) permit smaller sample sizes but demand stricter acceptance criteria (lower c_0), whereas narrower intervals (e.g., (0.2500, 0.3673)) require larger samples and allow a higher c_0. Across all tables, c_0 values range roughly from 0.47–0.66 for the widest interval to 0.26–0.29 for the narrowest. Furthermore, for a fixed value of m, an increase in the difference between c_{AQL} and c_{RQL} necessitates a smaller sample size for inspection, given the constraints of α-risk, β-risk, and c_{RQL} . The judgment process is comparatively more straightforward when the differences between c_{AQL} and c_{RQL} are more pronounced than when they are closely aligned [33].

Finally, variations in α and β reflect the trade-off between producer and consumer protection. Lower α (more stringent producer protection) increases n and tightens c_0, while higher β (greater tolerance for consumer risk) decreases n and relaxes c_0. By appropriately selecting m, α, β, and the (c_{AQL}, c_{RQL}) pair, practitioners can tailor the C_{pp}-RSP to balance inspection cost against quality assurance objectives in a rigorous, statistically controlled manner.

4.2 Average Sample Number (ASN)

Under quality level (c_{AQL}, c_{RQL}) = (0.5917, 1.0), and risk levels (α, β) = (0.01, 0.05), for the variable SSP (m=1) and the resubmitted sampling plan (m = 2, 3, 4), the ASN curves are shown in Figure 5. Figure 5 presents the ASN plot for both the fixed-sample C_{pp}-SSP (single-stage, VSS, m = 1) and the C_{pp}-RSP (sampling times, m = 2, 3, and 4). On the horizontal axis, the C_{pp} values vary from excellent (≈ 0.3) to poor (≈ 1.2). The vertical axis represents the average number of units inspected before a definitive accept or reject decision is made. The black horizontal line (m = 1) remains constant at ASN = n = 113, reflecting a predetermined sample size regardless of quality. In contrast, the red (m = 2), green (m = 3), and blue (m = 4) curves rise sharply in the mid-range of C_{pp}, where process performance is ambiguous, and flatten at lower ASN values for extreme quality levels. Specifically, for very low C_{pp} (< 0.6), indicating a high-quality lot, it can be identified soon without requiring many resubmissions, with an average of 60–80 inspections, and the m = 4 plan achieving the lowest ASN. As C_{pp} increases above the target, ASN also grows, depending on m, because more times accumulate inspections before acceptance. This plateau indicates that, despite its low quality, C_{pp}-RSP still requires additional inspections to ensure lot acceptability, due to the conservative nature of the resubmission system [19]. Thus, resubmitted sampling markedly reduces the average inspection effort at clear-cut quality extremes (high quality), when compared to the traditional plan (SSP) [23].

Moreover, Table 5 reveals a systematic interaction between the per-stage sample size n, the maximum number of sampling times m, and the worst-case Average Sample Number ASN_{max} across different risk settings (α, β) under (c_{AQL}, c_{RQL}) = (0.5917, 1.000). In line with Figure 5, for the design point (c_{AQL}, c_{RQL}) = (0.5917, 1.000) and (α, β) = (0.01, 0.05), Table 5 shows how increasing the maximum number of sampling times m reshapes both the per-stage sample size n and the worst-case inspection effort, ASN_{max}. When m = 1 (C_{pp}-SSP), the plan uses a fixed sample size n = 113, and the ASN is flat at ASN_{max} = 113, providing simple administration but no adaptation to lot quality. Allowing one resubmission (m = 2) reduces the required per-stage sample size n = 80, yet the worst-case ASN rises to about 159.88 because borderline lots may be inspected twice before a final decision is made. With two resubmissions (m = 3), n decreases further to 68, while ASN_{max} increases to roughly 203.76. For m = 4, the plan requires only 61 units per sampling stage but can reach a worst-case ASN of about 243.64. Thus, as m increases from 1

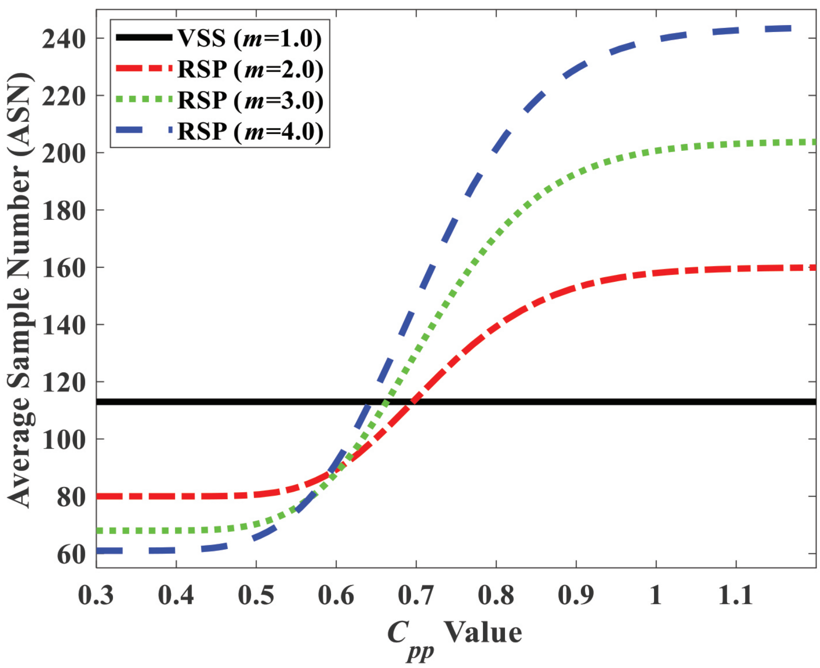


Figure 5. The ASN plot for C_{pp}-SSP (VSS) and C_{pp}-RSP with m = 2, 3, and 4 under (c_{AQL}, c_{RQL}) = (0.5917, 1.00), and (α, β) = (0.01, 0.05)

Table 5. The required n and ASN_{max} for C_{pp}-SSP and C_{pp}-RSP with different m under (c_{AQL}, c_{RQL}) = (0.5917, 1.000)

α	β	m = 1	m = 2		m = 3		m = 4	
		$n = ASN_{max}$	n	ASN_{max}	n	ASN_{max}	n	ASN_{max}
0.01	0.01	159	116	231.99	99	296.98	90	359.97
	0.05	113	80	159.88	68	203.76	61	243.64
	0.1	91	64	127.65	54	161.33	48	191.00
0.05	0.01	119	90	179.99	78	233.97	71	283.95
	0.05	80	59	117.85	51	152.72	46	183.58
	0.1	62	45	89.61	39	116.29	35	138.94
0.1	0.01	101	77	153.98	68	203.97	62	247.95
	0.05	65	49	97.84	42	125.69	39	155.57
	0.1	49	37	73.62	32	95.29	29	114.97

to 4, each individual inspection becomes "lighter" in terms of sample size, making it attractive for clear-cut good or bad lots, but the potential total number of inspected units in the most difficult, borderline situations can more than double relative to the SSP. This quantifies the core trade-off for practitioners at this parameter setting: higher m buys flexibility and reduced sample size per stage, at the cost of a taller ASN plateau and therefore higher possible labor and time requirements when lot quality lies near the acceptance–rejection boundary [14]. Overall, the table quantifies a clear trade-off: higher m yields leaner sampling per stage and higher efficiency for clearly good or bad lots, but it also pushes up the worst-case ASN, implying higher potential labour costs and slower decisions for lots whose quality lies near the acceptance–rejection boundary.

4.3 Operating Characteristics (OC) Curve

Figure 6 indicates the behaviour of the OC curve when $m = 1$ (i.e., the SSP), and the OC curve for the C_{pp}-RSP when $m = 2$, 3, and 4. Figure 6 illustrates the OC curves for the same four plans, showing the probability of lot acceptance versus C_{pp}. All curves are calibrated so that at C_{pp}=0.7909 (the AQL) the acceptance probability is about 0.95, and at C_{pp}=1.00 (the RQL) it falls to approximately 0.05. The single-sampling plan (black) features the steepest transition band, indicating a sharp discrimination between acceptable and unacceptable quality. The resubmitted sampling plans (red, green, and blue) progressively widen the "shoulders" of the curve as m increases, reflecting greater flexibility: when C_{pp} is far from decision thresholds, early stopping is likely, but when it is near these thresholds, additional stages smooth the accept/reject transition. Consequently, higher-stage C_{pp}-RSPs offer more gradual OC curves—trading decisiveness for adaptability—while still satisfying the same producer- and consumer-risk requirements.

Overall, C_{pp}-SSP ensures a straightforward and consistent inspection effort with sharp quality discrimination, but lacks flexibility. While the proposed C_{pp}-RSP offers dynamic inspection sizing, reducing the average sample size required to clearly identify good or bad lots, at the cost of potentially higher effort in ambiguous cases and increased administrative complexity [26]. The choice hinges on the trade-off between operational simplicity and efficiency gains across varying process capabilities [22, 23, 24].

Furthermore, from a methodological standpoint, the proposed C_{pp} RSP differs from bootstrap-based resampling schemes in that it relies on an explicit parametric model for the estimated C_{pp}, yielding analytic OC and ASN functions and closed-form risk-control constraints. This provides transparent plan characteristics, fast computation, and general tables that can be applied across processes that satisfy

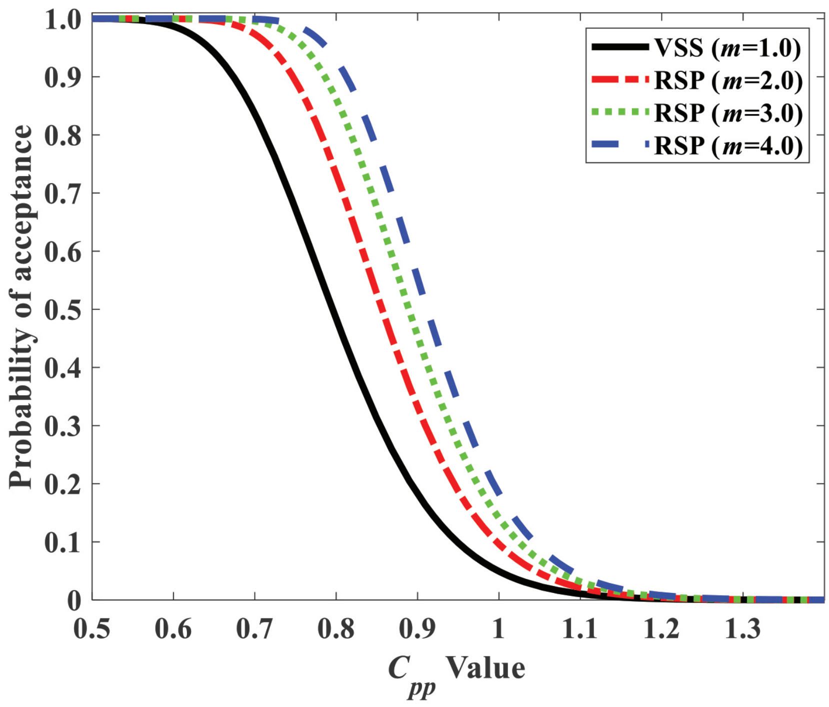


Figure 6. The OC plot for C_{pp}-SSP (VSS) and C_{pp}-RSP for $m = 2$, 3, and 4 with $(n, c_0) = (113, 0.7909)$

the normality assumptions. A bootstrap implementation, while attractive for handling non-standard distributions or limited data, would replace these analytic guarantees with simulation-based approximations and higher computational cost, and would need to be re-run for each new context.

5. Conclusions

This article develops a revised sample approach for variables to mitigate the issue of lot sentencing, using the process incapability index. An examination of the suggested sampling plan's performance in diverse situations. If $m = 1$, the resubmitted sampling plan simplifies to the standard SSP for variables. The study demonstrates that the proposed C_{pp}-RSP markedly enhances inspection efficiency by reducing the ASN at clear-cut quality extremes while rigorously maintaining producer's and consumer's risk levels commensurate with a fixed-sample SSP; however, the increased worst-case inspection burden in marginal quality regions and the administrative complexity introduced by multiple decision points represent its principal limitations. The strengths of the C_{pp}-RSP include its early-stop capability, adaptability to process variability, and preservation of α/β protections. Its weaknesses, however, involve the potential for a higher plateau ASN under ambiguous process capability and the need for robust logistical support to manage repeated sampling times. Furthermore, for practical purposes, tables containing the plan parameters under different scenarios are supplied. The tables enable professionals to comprehend the necessary sample size for inspection and the corresponding critical value that aids in decision-making.

Finally, this article focuses on analytical tractability, a parametric design for C_{pp}-RSP, assuming normality to derive closed-form OC and ASN functions. It is important to note that ignoring the normality assumption can lead to inaccurate assessments of delivered lots, potentially resulting in unreliable lot decisions. Moreover, a bootstrap-based resampling design is suggested as a future extension for non-normal or complex processes. Future research can integrate bootstrap resampling to assess robustness under model misspecification and to compare computational cost and risk-control properties. Additionally, an economic analysis can be conducted by embedding C_{pp}-RSP into existing cost models, specifying context-dependent post-rejection policies and associated unit costs (accept under concession/downgrade, 100% rework/repair, scrap, and return to supplier). The last extensions can address the dynamic optimization of the resubmission number m and the hybridization of the C_{pp}-RSP with other variable sampling schemes to jointly minimize total inspection cost while preserving the desired OC and ASN characteristics.

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