



AN INVERSE OPTIMIZATION PROBLEM OF HEAT TRANSFER IN THE MACHINING PROCESS – A REVIEW

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Abstract: This paper reports on the results of research on thermal aspects in the process of material removal by inverse heat transfer problem. The research focuses on the identification, modeling and optimization of machining process based on the measured temperature at a particular point of the workpiece. The inverse approach determines the overall temperature distribution of the workpiece and the unknown heat flux at the tool/workpiece interface in machining. By introducing and minimizing an objective function based on the heat flux function, relationship of the heating power and duration on the surface layer of the workpiece is optimized. In this way, the most favourable machining conditions are determined in order to achieve high productivity and quality levels. The inverse optimization problem is solved by using the analytical, numerical and regularization methods. Formulation, application and analysis of the inverse optimization problem of heat transfer are shown on the example of creep-feed grinding. The creep-feed grinding process is a widely used abrasive machining process that is characterized by high thermal load of the workpiece. The results of the inverse optimization problem were verified by a series of experiments under different machining conditions.

Key words: Temperature, Thermal energy, Inverse approach, Minimization technique, Creep-feed grinding

Problem inverzne optimizacije prenosa toplote kod obradnog procesa - pregled. Ovaj rad prikazuje rezultate istraživanja toplotnih aspekata procesa obrade skidanjem materijala inverznim problemom prenosa toplote. Istraživanje se fokusira na identifikaciju, modeliranje i optimizaciju procesa obrade na osnovu izmerene temperature u određenoj tački obratka. Inverzni pristup određuje ukupnu raspodelu temperature obratka i nepoznati toplotni fluks u zoni kontakta alat/obradak. Uvođenjem i minimiziranjem objektivne funkcije zasnovane na funkciji toplotnog fluksa, optimizuje se odnos snage i vremena zagrevanja površinskog sloja radnog predmeta. Na taj način se određuju najpovoljniji uslovi obrade, kako bi se postigla visoka produktivnost i kvalitet obrade. Problem inverzne optimizacije rešava se analitičkim, numeričkim i regularizacionim metodama. Formulacija, primena i analiza problema inverzne optimizacije prenosa toplote prikazana je na primeru dubokog brušenja. Proces dubokog brušenja je široko korišćeni abrazivni postupak koji se odlikuje velikim toplotnim opterećenjem radnog predmeta. Rezultati inverznog problema optimizacije su verifikovani sa nizom eksperimenata izvedenih pod različitim uslovima obrade.

Кljučне речи: Temperatura, Toplotna energija, Inverzni pristup, Tehnika minimizacije, Duboko brušenje

1. INTRODUCTION

Material removal process is a fundamental manufacturing technology used in a great variety of industrial applications during which various products are designed, manufactured and assembled. This technology is widely used in order to produce high quality mechanical parts from many different material categories, such as metal, plastic and ceramic. A key feature of the process of material removal is his ability to produce vital components of complex geometry, exceptional surface finish, high dimensional accuracy, and usually from difficult-to-machine materials. It covers a wide range of material removal operations such as turning, drilling, milling, grinding and other processes [1-4].

On the other hand, the primary disadvantage of the material removal process is development of a large amount of thermal energy within a relatively narrow zone of chip formation. The generated thermal energy and the problem of its removal from the machining area initiates the development of high cutting temperature.

This high cutting temperature has a pronounced negative effect on the tool life, and dimensional accuracy, surface integrity and metallurgical transformations of the workpiece material [5-9].

In advanced manufacturing technology, grinding is considered one of the most important processes of material removal and in some way it is involved in the production of almost every mechanical product. Applications and advantages of grinding process include good surface finish and correct dimensional tolerances. In accordance to that, grinding is used in high-precision work, machining of complex surfaces, but also for hard-to-machine materials. Basic grinding operations are cylindrical grinding, surface grinding, centerless grinding, profile grinding and etc [10, 11].

In recent years, in addition to conventional grinding, high-performance grinding has been introduced [12]. The conventional grinding uses high workpiece speed and very small depth of cut (multi-pass grinding). The concept with high-performance grinding is to increase cutting speed (high-speed grinding) or/and to slow the workpiece speed to allow increase of the depth of cut

(creep-feed grinding) [6, 9]. The emphasis in high-performance grinding is to increase low productivity which is considered the main disadvantage of conventional grinding. However, the high-performance grinding conditions considerably change the geometry and kinematics of the cutting process that leads to more intensive development of thermal energy in the contact zone [13-15]. Extremely high heat is a basic constraint for the further development of high-performance grinding.

Scientific research in the field of material removal process has yielded a number of approaches to the study of thermal effects of machining. The earliest works focused on the theoretical and experimental determination of machining temperature [16]. A review of theoretical and experimental methodology for determination of machining temperature has been shown by Barrow [17] and more recently by other authors [6, 8, 18]. Later works were carried out from the analytical perspective of the thermal aspects of material removal processes [19, 20]. The analytical research of machining temperature distribution, using Jaeger's classical solutions of moving heat sources [21], has been presented by Chao and Trigger [22] and Shaw [23]. A large number of numerical methods were introduced in the early 1980s. [11]. Ostafiev et al. [24] investigated a numerical heat models in continuous and interrupted cutting. Zhang et al. [25] provide a recent study of the finite element analysis of temperature distribution based on a heat source in the grinding process. Finally, the development and application of artificial intelligence, advanced and hybrid solutions of the thermal aspect of machining followed at the end of the 20th century and at the beginning of the 21st century. Kovac et al. [26] designed a fuzzy logic system to model the relationship between the machining conditions and surface roughness, respectively cutting temperature. Gostimirovic et al. [27] have utilized the intelligence multi-objective algorithm for optimization of energy in non traditional machining processes. Rodic et al. [28, 29] developed advanced models that can be used for the prediction of machining energy during the material removal processes. Application of hybrid techniques in thermal behaviour of the machining process has been shown by Grzesik [30].

Since the main problem of material removal process is to achieve a high-quality finish with the highest possible productivity level, special attention is focused on the optimization of the processing conditions. There is a series of works which deal with optimization of machining strategy using various techniques [3, 4, 27, 31-33]. However, although heat and temperature are parameters which directly affect the process performance, their use for the optimization is quite complex in nature. For the above reasons, constant efforts are being made to improve existing and develop new optimization approaches of thermal phenomena in machining process.

As previous research has shown, non-stationary and non-linear machining process that involve intensive heat transfer, can be successfully solved using an inverse heat transfer problem. The inverse heat transfer problem is today applied in design, modeling and optimization of

thermal systems. In the case of material removal process, the inverse problem has so far been mainly used to identify process by means of heat flux or temperature field in the machining zone. Recently, there are more papers that investigate the thermal effect of the machining process based on the inverse problem [34-36]. Gostimirovic et al. [37-40] used the inverse heat transfer problem as a way for the optimization of machining.

The theoretical basics of the inverse heat transfer problem were given by Tikhonov and Arsenin [41], Beck et al. [42] and Alifanov [43]. Ozisik and Orlande [44] and Colaco et al. [45] presented the new scientific approaches of the inverse modeling and optimization problem in heat transfer. It has been shown that the inverse problem of heat transfer allows the closest possible approximation of thermal performance. However, how the inverse problem is classified as an ill-posed, there is no single and stability solution to this problem [41-43]. The uniqueness of the solution is ensured by experimental design and in order to satisfy the stability condition special solution techniques are required. Perakis and Haydn [46] indicate the influence of temperature measurement error in the inverse determination of heat flux. A stability analysis of the inverse problem of heat conduction was given by Gostimirovic et al. [40].

The research presented in this paper deals with the optimization of the machining process using an inverse heat transfer problem. The machining process is used in the complex engineering components industry, where the final product is obtained by removing excess material from the workpiece. As the main task of the machining is to achieve high productivity with good quality, special attention is directed on the aspects of thermal energy transfer. If the machining conditions are poorly defined, the thermal effects can significantly reduce the product performance characteristics. Since it is a very complex heat transfer in a narrow area, inverse optimization problem has proved to be well suitable for the investigation of thermal aspects of the machining process. For the temperature measured at any point of the workpiece, the temperature and heat flux in the machining zone can be determined by convergent numerical methods. Then by minimizing the objective function using an iterative procedure the optimal machining parameters can be set. Verification of the solution of inverse optimization problem of heat transfer was carried out during creep-feed grinding which is distinguished by extreme thermal conditions. The calculations showed a good agreement with the experimental data, which confirms the reliability of the inverse optimization approach.

2. THERMAL ASPECTS OF CREEP-FEED GRINDING

Creep-feed grinding is an abrasive machining operation employed for removal of the difficult-to-machine materials, particularly where precise form is required on the finished part. Creep-feed grinding allows much higher material removal rate, shorter cycle time, lower grinding wheel wear and good surface

roughness [9, 37]. Creep-feed grinding is characterized by low workpiece speed and high depth of cut resulting in a large length of contact between the grinding wheel and workpiece. In that context, the prolonged time of contact wheel/workpiece, results in the generation of intense heat release on the surface layer of the workpiece material when compared with multi-pass conventional surface grinding, Fig. 1.

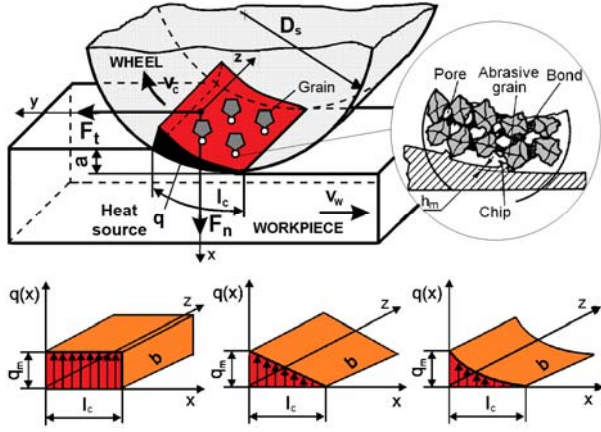


Fig. 1. Thermal model of creep-feed grinding

2.1 Grinding mechanism

As already stated above, in creep-feed grinding, very slow (creep) traverse feed rate of wheel head table and very heavy depth of cut is used. For that reason, creep-feed grinding mechanism is implemented with a very large number of abrasive particles on a grinding wheel that remove a thick layer of the workpiece material, and that is inserted in the space pores between the abrasive particles and the bond of the grinding wheel. When the grinding wheel comes out of the workpiece material, a large amount of chips leaves hardly the pores of the grinding wheel under the influence of centrifugal force or the medium for cooling and flushing. Due to increased friction, but also because of a large contact length between the grinding wheel and workpiece material, mechanical and metallurgical changes in the surface layer of the workpiece material are possible due to the high heat energy in the cutting zone [14, 15, 35].

The process of creep-feed grinding is defined by the geometric and kinematic parameters. The geometric parameters are: wheel diameter D_s , depth of cut a , specific material removal rate Q'_w , length of contact l_c and average chip thickness h_m . The kinematic parameters are: wheel cutting speed v_s , workpiece speed v_w , and grinding speed ratio s .

The length of contact arc in surface creep-feed grinding is the active distance that the abrasive particle exceeds from the moment of contact with the workpiece material to the exit from the contact:

$$l_c = \sqrt{a \cdot D_s} \quad (1)$$

The average chip thickness is approximately half of the average chip thickness cut with abrasive particle, and is determined by:

$$h_m = \frac{a \cdot v_w}{v_s} = \frac{a}{s} = \frac{Q'_w}{v_s} \quad (2)$$

In the Eq. (2) is noticed grinding speed ratio, which defines how often an abrasive particle passes over the same place on the workpiece material:

$$s = \frac{v_s}{v_w} \quad (3)$$

and specific material removal rate, which shows the amount of the volume of material removed from the workpiece per grinding wheel width in unit time:

$$Q'_w = a \cdot v_w \quad (4)$$

As a result of plastic deformation in the grinding zone and friction between the abrasive particles and workpiece material, cutting force occurs. The resulting cutting force is the result of the simultaneous action of the active abrasives which process the workpiece material. For surface creep-feed grinding, the resulting cutting force has been regrouped on the tangential and normal components. The tangential force acts in the direction of the cutting speed, and the normal force acts normally to the contact surface of the grinding wheel and workpiece material.

An empirical equation of the tangential grinding force by Kronenberg is determined as follows [1]:

$$F_t = A \cdot k_s \quad (5)$$

where $A=l_c \cdot b_c$ is the grinding contact area (b_c is the width of grinding) and $k_s=h_m^{-p} \cdot k_{s1.1}$ is the specific cutting force (h_m is the average chip thickness and $k_{s1.1}$ is the nominal specific cutting force for workpiece material).

According to the relation between tangential and normal force [47, 48], the normal grinding force can be indicated as:

$$F_n = \frac{F_t}{\mu} \quad (6)$$

where μ is the friction coefficient.

On the basis of the tangential grinding force, specific grinding energy can be expressed [49], which defines energy consumed per unit volume of the material removed:

$$e = \frac{P}{Q'_w} = \frac{F_t \cdot v_s}{a \cdot v_w} = \frac{F_t}{h_m} \quad (7)$$

where $P=F_t \cdot v_s$ is the grinding power.

2.2 Heat performance of grinding

From the above discussion, it is evident that creep-feed grinding leads to the generation of large amount of thermal energy in the region of the cutting zone [14, 15, 37]. The generated thermal energy and the problems of its removal from the grinding zone lead to the development of high cutting temperatures. These elevated temperatures have a pronounced detrimental effect on the grinding wheel and the workpiece material. Thereby, the determination of the thermal properties of creep-feed grinding largely depends on the knowledge of basic mechanisms of heat generation and transfer and the characteristics of the temperature

distribution in the cutting process. In the current research, it is becoming increasingly popular an integrative mathematical-experimental investigation of the thermal aspects of creep-feed grinding.

The amount of heat energy generated in the grinding process is equivalent to mechanical work required to machining, Fig. 2. Here's almost all of the mechanical work transformed into heat by deformation (Q_d), friction (Q_f) and separation (Q_s). Thus generated heat is evacuated from the cutting zone over the coolant (Q_c), workpiece material (Q_w) and grinding wheel (Q_{wh}) and environment (Q_e). The heat is transferred by conduction, convection or radiation [10].

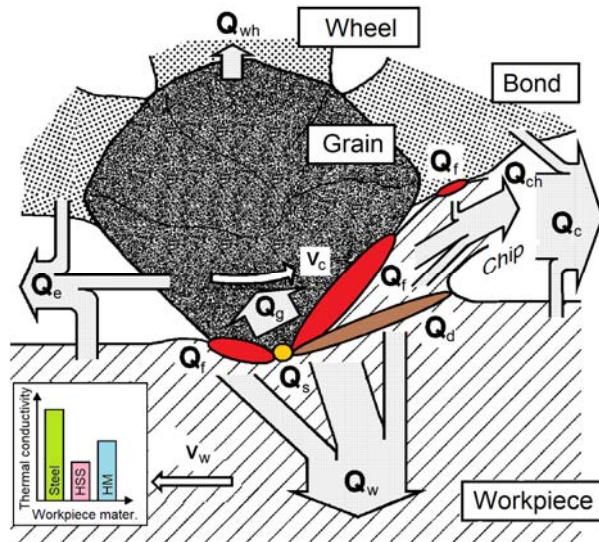


Fig. 2. Heat generation and distribution in the grinding

The majority of the generated heat in creep-feed grinding should be distributed through the coolant. However, inadequate grinding conditions can lead to transfer of large amounts of heat over the workpiece and cause serious changes in the surface layer of the workpiece material (microstructure, microhardness, residual stress, crack and burn). These thermal changes of the material are called heat affected zone [6, 9]. If dimensional errors appear due to the thermal deformation, the grinding thermal load reduces the exploitation of the final product.

Similar to other studies of thermal processes, research into the nature of thermal energy in creep-feed grinding requires the development of an adequate model for certain behaviour. As the performance of thermal grinding process depends on a number of influencing factors, it is very difficult to define the thermal model of the grinding zone. The previous Fig. 1 shows a schematic diagram of a simple creep-feed grinding thermal model.

When grinding, it can be assumed that an elemental heat source of abrasive particle is the result of deformation and friction between the abrasive and the workpiece material. By summing all the elemental heat sources, the total heat source for the entire grinding zone is obtained. This heat source acts continuously, moves over the workpiece surface at a constant speed, and is classified as an internal heat type. The shape and

dimensions of the heat source depend on the grinding wheel/workpiece interface. The distribution law of the heat source is very difficult to predict, and in the thermal analysis of the grinding process rectangular, triangular and exponential profiles are usually encountered.

At the grinding process, if taken into account that the amount of heat is equivalent to mechanical work [1], then follows:

$$Q = F_t \cdot v_s \cdot t \quad (8)$$

where t is the heat transfer time.

The amount of heat is transformed into heat flux, and using Eq. 5 follows the expression:

$$q \cong \frac{Q}{A \cdot t} = \frac{F_t \cdot v_s}{A} = v_s \cdot k_s \quad (9)$$

By substituting Eqs. (2) and (5) in Eq. (9), the final expression for the heat flux at the grinding is:

$$q = v_s \cdot h_m^{-p} \cdot k_{s11}^{-p} = v_s^{1+p} (a \cdot v_w)^{-p} \cdot k_{s11} \quad (10)$$

On the other hand, the heat transfer time in the grinding is expressed as the ratio between the length of contact l_c and workpiece speed v_w , that is:

$$t = \frac{l_c}{v_w} \sqrt{\frac{a \cdot D_s}{v_w}} \quad (11)$$

Therefore, Eq. (10) and (11) define the heat source power and time of the heat source (heating power and duration) in the creep-feed grinding process.

The heat distribution in grinding depends greatly on the machining conditions and physical properties of the workpiece material, grinding wheel and coolant. Considering the performance of the creep-feed grinding, a part of the heat transferred through the workpiece material is of the most importance. The amount of heat that reaches the workpiece is defined by an energy partition, and can be expressed with the equation:

$$p(e) = \frac{q_w}{q} \quad (12)$$

where q_w is the heat transferred into the workpiece material.

Most of the researches, as a foundation for analytical determination of the energy partition, use the Green's function of point heat [21]. One of the ways for determining the energy partition has the following form [50, 51]:

$$p(e) = \frac{1}{1 + \sqrt{\frac{(k\rho)_c \cdot v_c}{(k\rho)_w \cdot v_w}}} \quad (13)$$

where $k\rho$ is physical properties of the grinding wheel and workpiece material (k is the thermal conductivity, ρ is the material density, c is the specific heat capacity).

3. INVERSE OPTIMIZATION OF GRINDING

The investigation of any system or process generally is considered from the standpoint of relations between the input and output parameters. This involves

modeling the real problem that is valid in a particular domain of activity into consideration. Form of a mathematical model, if the input and process parameters are given, and the output is found, is defined as a direct (forward) problem introduced by Hadamard in 1923 [52]:

$$y := \mathbf{A}(z)x \quad (14)$$

where $x \in X$ is the input parameters (X is the space of input values), $y \in Y$ is the output parameters (Y is the space of output values) and \mathbf{A} is the process operator from X into Y associated to the parameter identification problem $z \in Z$ (Z is the space of process variables).

From other side, if the process and output parameters are given, and the input is found, it is an inverse problem defined as:

$$\mathbf{A}(z)x = y \quad (15)$$

It is well known that the solution of a well-posed direct problem must satisfy the condition of stability and uniqueness with respect to the process parameters. However, in the inverse problem, stability of the solution depends on the model used and errors in the input data, and unique solution does not exist. The problem can still be solved with the help of additional information about the model, so the problem is solved as follows:

$$x := \mathbf{A}^{-1}(z)y \quad (16)$$

where \mathbf{A}^{-1} is the inverse operator of the problem.

The inverse problem is scientific method dealing with inversion of model parameters. It is a mathematical procedure that allows the correlation of unknown input and known output data, with status display of the process. Thus, the inverse problem is used to obtain causal information from observed effects, and which cannot be directly observed from conducted measurements. The inverse problem is encountered in many fields of application, and is used as an acceptable approach for the identification, design and optimization.

3.1 Inverse heat transfer approach

Research of heat transfer performance is a very significant process in the modeling and analysis of any thermal system or process. An inverse heat transfer problem is a novel approach that describes well a mathematically ill-posed process of intensive heat exchange. Above all, the inverse heat transfer problem has been associated with the evaluation of unknown boundary heat flux or boundary temperature conditions appearing in the mathematical description of the problem. However, in order to find a solution to this problem, additional information must be ensured by measuring the temperature within the observed region.

Modeling of the inverse heat transfer problem starts from the development of model for a limited domain with the well-known boundary and additional conditions. The actual thermal process is most often described by a mathematical model. In doing so, the aim is to set up an adequate mathematical model, but in as simple form as possible to facilitate the solution. In the literature, a reduced modeling of the thermal

process is very popular, because it simplifies research of a complex actual process. Now, by applying the appropriate mathematical technique, the model defined in this way is solved quickly and efficiently.

In grinding, since the depth of cut is many times less than the length of the wheel/workpiece interface, the heat source can be approximated by a series of adiabatic plates, Fig. 3. Further, if heat dissipation in the direction of movement of the heat source is neglected, then the workpiece can be treated as a plate of infinite length and constant heat distribution. Replacing the workpiece with a semi-infinite plate is justified, given that the heat source during grinding is created in a small volume of workpiece material. On the other hand, the assumption of constant heat distribution in the interface is a good approximation in the case of heating the surface layers of the workpiece material.

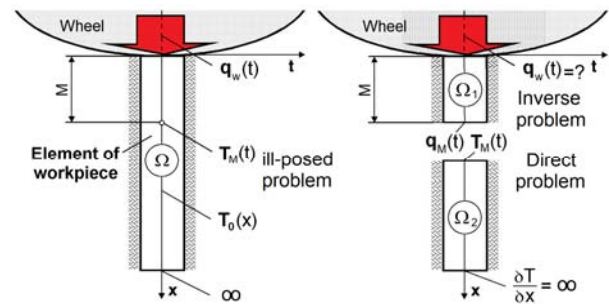


Fig. 3. Inverse heat transfer modeling of grinding

The mathematical model of the thermal process is most often described by a system of differential and algebraic equations, since such a form is efficiently calculated on a computer. For a defined thermal model of grinding, the following is a general case of differential equation of a one-dimensional inverse heat transfer problem ($x \in (0, \infty)$ and $t \in (0, t_m]$):

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (17)$$

where $T=T(x,t)$ is the temperature at coordinate x at time t , $\alpha=k/\rho c$ is the thermal diffusivity and t_m is the largest time increment.

The initial condition is the temperature distribution at the starting time:

$$T(x, t) \Big|_{t=0} = T_0 \quad (18)$$

The lower boundary condition for the surface layer of the workpiece is defined by the boundary condition of the second order, as the known heat flux:

$$-k \frac{\partial T}{\partial x} \Big|_{x=\infty} = 0 \quad (19)$$

where k is the thermal conductivity.

The additional condition is provided as the measured temperature at a point on the workpiece $T_M=T(x,t)$, where is the $0 < M \leq \infty$:

$$T(x, t) \Big|_{x=M} = T_M \quad (20)$$

The solution to the inverse heat transfer problem thus posed represents the determination of the workpiece temperature distribution $T_\Omega=T(x,t)$ in whole

domain $\Omega = \{(x, t): x \in (0, \infty), t \in (0, t_m)\}$ and the unknown boundary heat flux to the workpiece $q_w = p(e) \cdot q$, respectively:

$$\begin{aligned} T(x, t)|_{\Omega} &= T_{\Omega} \\ -k \frac{\partial T}{\partial x} \Big|_{x=0} &= q_w(t) \end{aligned} \quad (21)$$

The inverse heat transfer problem, classified as an ill-posed problem, can be solved by dividing the set assignment (Eq. 17 to 21) into two separate steps: the ill-posed inverse problem (domain $\Omega_1 = \{(x, t): x \in (0, M), t \in (0, t_m)\}$) and the well-posed direct problem (domain $\Omega_2 = \{(x, t): x \in (M, \infty), t \in (0, t_m)\}$).

The direct problem can be analyzed as a mathematical well-posed problem, since there are known boundary conditions in the domain Ω_2 . The temperature distribution $T_{\Omega_2} = T(x, t)$ and the heat flux q_M at the point M are the solution to the well-posed problem.

On the other hand, the inverse problem in the domain Ω_1 is analyzed as a mathematical ill-posed problem, since the boundary condition on the workpiece surface is unknown. However, the determined heat flux q_M can be related to this domain, so there are two conditions at $x=M$ and neither one at $x=0$. In this context, inverse problem is solved starting from the known boundary conditions at the lower limit. The temperature distribution $T_{\Omega_1} = T(x, t)$ and unknown heat flux boundary condition on the workpiece surface q_w are the solution to the ill-posed problem.

The ill-posed inverse heat transfer problem can be solved by using the different approximation methods (analytical, numerical, regularization, etc.) [41-45].

As for the analytical approach, the Green's function for a point heat source is generally accepted as the basic for solving the inverse heat transfer problem [21]. If an insulated heat source is assumed to be used, the following is the expression for the heat flux and temperature distribution:

$$T(x, t) = \frac{q}{k} \sqrt{\frac{\alpha}{\pi}} \int_0^{t_m} \frac{1}{\sqrt{t_m - t}} e^{-\frac{x^2}{4\alpha(t_m - t)}} dt \quad (22)$$

Due to the great complexity of the partial differential equation which describes the process of heat transfer during grinding, it is mostly solved by the numerical methods: finite difference method or finite element method. These methods transform the differential equations into systems of approximate algebraic equations by discretization. The discretization implies the approximation of the adiabatic plate of the workpiece to a certain number of elements Δx . Since it is about a non-stationary heat transfer problem, time change of temperature and heat flux is discretized with Δt . Thereby, using the computer during solving the numerical methods the high accuracy of the solution can be achieved.

The concept of the finite difference method is clear and simple, because it follows the physical basis of the process, where the temperature at a certain node is calculated as a result of heat exchange with adjacent

nodes. If the method of explicit finite difference is chosen, based on three known temperatures at adjacent nodes, the temperature and the heat flux at the next moment are calculated through the following expression [38, 43]:

$$\{\mathbf{T}\}^{t+\Delta t} = [\boldsymbol{\alpha}] \cdot \{\mathbf{T}\}^t + \{\mathbf{q}\} \quad (23)$$

where $\{\mathbf{T}\}$ is the temperature vector of discrete nodes, $[\boldsymbol{\alpha}]$ is the thermal diffusivity matrix and $\{\mathbf{q}\}$ is the heat flux vector.

The finite element method is based on variational principle to obtain the approximate solution, where the method of integrating the differential equation is replaced by the minimum value of the integral [53]. By using the finite element method, the observed domain is divided into a finite number of nodes, and the variational problem is transformed into a nonlinear mathematical program with multiple constraints. The differential equation is solved by solving algebraic equations, it is very easy to use and has high accuracy in calculation. There are several methods that are used for solving the unknown temperature and heat flux at the next time step. Galerkin's method that is used to solve partial differential equation of heat transfer can be expressed in matrix form as:

$$[\mathbf{k}] \cdot \{\mathbf{T}\}^t + [\mathbf{c}] \frac{\partial \{\mathbf{T}\}^{t-\Delta t}}{\partial t} = \{\mathbf{q}\} \quad (24)$$

where $[\mathbf{k}]$ is the heat conductivity matrix and $[\mathbf{c}]$ is the specific heat matrix.

The solution of the inverse heat transfer problem can be successfully realized by applying the method of regularization [41]. This is a method of solving ill-posed problems that translates a differential equation into solving arbitrary systems of linear algebraic equations. Thereby, the method of regularization means that an inverse problem solution is obtained with the help of approximate methods which allow control across the measure of proximity of a calculated solution as compared to an exact solution one by changing the usual computational algorithms. Such an approach to inverse heat transfer problem solution is very popular due to the simplicity of the computational algorithms used for their realization. Besides, the inverse problem solution by the regularization is more appropriate to overcome the instability that results from noisy measurements of input parameters, because in this case the input values are given by the desired state.

There are different methods in use for defining the regularizing algorithm. A variational method based on the minimization of an objective function is the most used [41, 43]. In order to treat the inverse heat transfer problem in the form of an optimal control, the regulation heat flux q is performed from the conditions of conformity of the known temperature T_M with the calculated temperature T_q in the space of objective function U :

$$q(t) = \arg \inf \Theta_U(T_q(q, M, t), T_M(t)) \quad (25)$$

where Θ_U is the measure of proximity (distance between values).

3.2 Inverse optimization technique

The use of inverse optimization techniques represents a new research paradigm. In the recent past, inverse optimization problem has evolved from a theoretical research topic to a practical method of engineering analysis. Despite similarities, inverse problem and inverse optimization problem are conceptually different. Inverse problem is concerned with the identification of unknown input values, and optimization problem deals with the minimization or maximization of a certain objective function, in order to find optimal parameter values. The solution methodologies of a single-objective inverse optimization problem are as a rule based on minimization techniques (gradient, stochastic, and heuristic methods).

For the solution of inverse optimization problem by the minimization techniques, the first step is the definition of an objective function. The objective function is the mathematical representation of an aspect to be optimized, and it can be mathematically stated as:

$$U = U(x) \quad (26)$$

where x is the variable of the problem that can be modified in order to find the minimum value of the objective function U .

For inverse optimization problem, the objective function usually involves the squared difference between measured and calculated variables of the process under consideration. For that matter, the objective function that provides minimum variance estimates is the sum of the squared residuals defined as [42]:

$$U(x) = [Y - T(x)]^T [Y - T(x)] \quad (27)$$

where Y and $T(x)$ are the vectors containing the measured and calculated temperatures, respectively.

If the inverse problem of heat transfer involves the calculation of the unknown heat flux q_w , this is usually solved using the procedure called Tikhonov's regularization which minimizes the least squares norm [41]. This can be mathematically stated as follows:

$$U(q_w) = \sum_{j=1}^N [T_q(q_w^j, M, t_j) - T_M(t_j)]^2 \quad (28)$$

where T_q is the calculated temperature at point M based on function of the process, T_M is the measured temperature at time $t \in (0, t_N]$ and N is the number of measurements.

There are a number of minimization techniques for a particular objective function [41, 43]. Basically, this type of minimization problem in an infinite dimensional functional space is solved by an iterative procedure, which after a certain number of iterations converges to a minimum of the objective function. The iterative procedure can be written in the following form:

$$q_w^{k+1}(t) = q_w^k(t) + \beta^k \cdot d^k \quad (29)$$

where q_w is the initial value of design variable (heat flux), β is the search step size, d is the direction of descent and k is the iteration number.

The iterative procedures that converge to a local minimum with search directions defined by a gradient are called the gradient method, Fig. 4. The primary gradient methods are gradient descent and conjugate gradient. The basic idea of these methods is to find the true minimum value of the objective function U , where the direction of descent and the search step size are given by, respectively:

$$d^k = -\nabla U(q_w^k)$$

$$\beta^k = \frac{\|\nabla U(q_w^{k+1})\|^2}{\|\nabla U(q_w^k)\|^2} \quad (30)$$

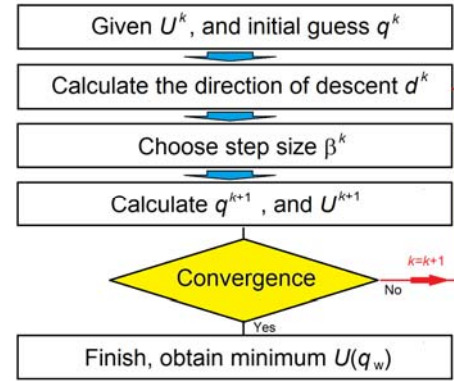


Fig. 4. The gradient iterative method of minimization

The iteration step is acceptable if objective function $U^{k+1} < U^k$. In doing so, the iterative procedure is repeated until the minimization of the objective function is achieved. This procedure is considered to have converged for a sufficiently small error rate, if the calculated and measured temperatures are basically almost the same:

$$\min_q U(q_w) \quad (31)$$

4. TESTING PROCEDURE

Analysis of the inverse problem of optimizing the heat transfer was carried out in the case of creep-feed grinding. Based on the discussion so far, it is evident that in creep-feed grinding a substantial amount of thermal energy is generated. Since the basic task of the creep-feed grinding is to achieve the high productivity and machining quality, justified attention is focused on the effect of heat on the exploitation characteristics of the workpiece surface.

4.1 Experimentation

The experiment was conducted on the surface creep-feed grinding machine Majevisa CF 412 CNC from Republic of Serbia. The high porosity Winterthur grinding wheel 53A80 F15V PMF, dimensions 400×50×127 mm, was selected. The experiment was conducted with sharp grinding wheel. The workpiece material used in the work was the high speed tool steel (HSS), by the designation DIN S 2-10-1-8 (W.Nr. 1.3247). The surface hardness was the range 66±1 HRC, and dimensions of the specimen 40×20×16 mm.

The machining conditions were the depth of cut $a=0.5$ mm and the workpiece speed $v_w=5$ mm/s, i.e. the specific material removal rate $Q_w^1=2.5$ mm³/mm s. The grinding wheel cutting speed was constant, $v_c=30$ m/s. The coolant emulsion 6 % was used, the flow rate of 175 l/min and pressure of 25 bar.

The creep-feed grinding temperature was measured using a type K thermocouple, which can read temperatures in the range of -270 to 1370 °C. The chromel and alumel wires of the thermocouple hot junction of $\phi 0.2$ mm are insulated by a ceramic tube 1.6 mm outer diameter with two holes 0.4 mm inner diameter. The thermocouple was placed in a hole on the workpiece 1.8 mm diameter, touching the bottom of the hole which was at a distance $x=4$ mm from the grinding wheel/workpiece interface. During the experiment, after passage of the grinding wheel above the thermocouple junction, the corresponding temperature of the surface layer of the workpiece is registered. Measurement of the temperature in the creep-feed grinding was done with a measuring system, Fig. 5.

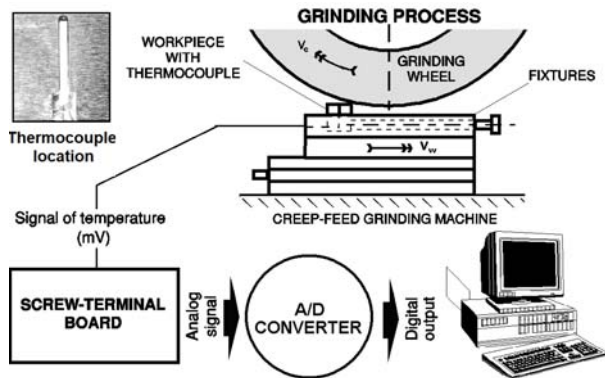


Fig. 5. Temperature measurement system in grinding

The results of measuring the creep-feed grinding temperature are presented in Fig. 6. The diagram shows the changes of workpiece temperature with time, at the different distances of the measuring points from the contact surface of the grinding wheel/workpiece interface, $x=0$ to 3.5 mm. After the passage of the grinding wheel over the thermocouple junction ($x=0$ mm), the maximum surface layer temperature is registered. From the diagram it can be concluded that the creep-feed grinding temperature is characterized by high value and short duration of action.

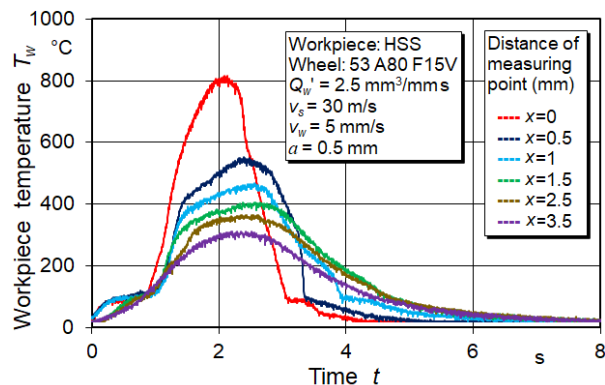


Fig. 6. Workpiece temperature at creep-feed grinding

Based on the grinding temperature distribution measured during each pass of the grinding wheel (Fig. 5), the temperature change in the surface layer of the workpiece material can be determined. In Fig. 7 is shown the temperature change by depth of the workpiece for different time periods, $t=0$ to 5 s.

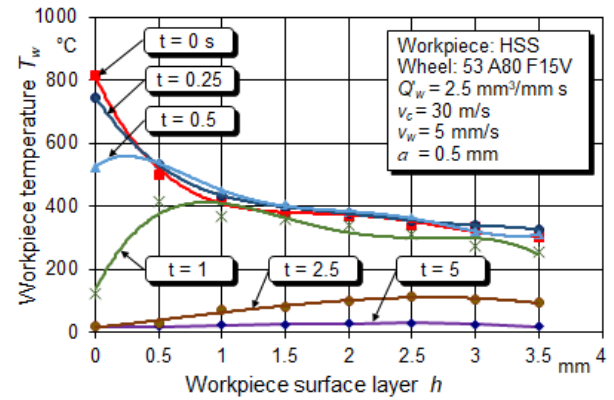


Fig. 7. Creep-feed grinding temperature distribution in the workpiece surface layer

4.2 Inverse modeling and optimization

Inverse modeling and optimization of the heat transfer in creep-feed grinding is performed with the use of mathematical methods and experimental results. The main goal is to determine the grinding temperature and heat flux distribution, as well as the most favourable performance of the heat flow, such as the heating power and duration.

Fig. 8 illustrates the inverse heat transfer modeling and optimization of the creep-feed grinding process.

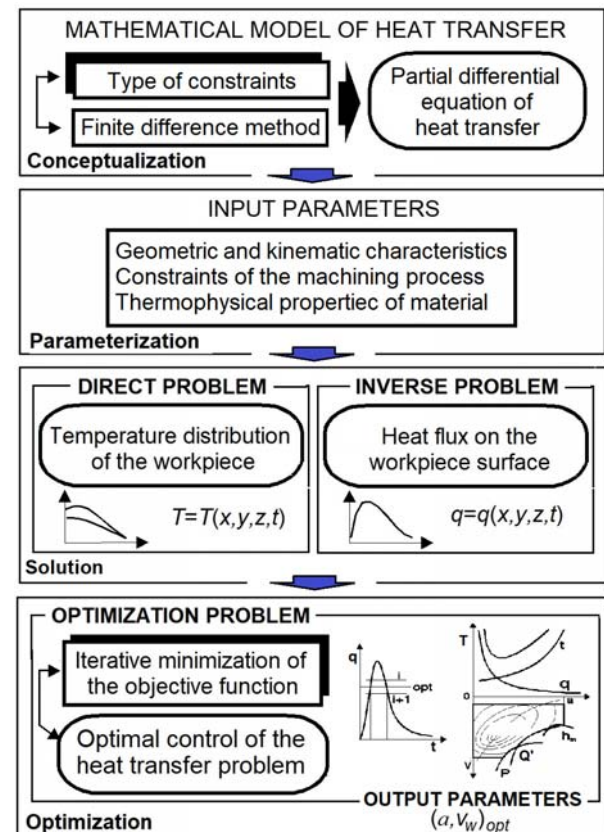


Fig. 8. Inverse heat transfer optimization method

The inverse optimization problem is developed in three steps. The first step (conceptualization) is to define a mathematical model of the heat transfer in the wheel/workpiece interface: partial differential equation for a limited region, type of constraints and way of solving the problem. The second step (parameterization) sets the input parameters: grinding geometric and kinematic parameters, values of the constraints and thermophysical properties of the materials. The third step (optimization) minimizes the objective function to find the maximum effects.

In this case of verification, the inverse heat transfer problem of the grinding is described by the simplified of Eq. (17) to (21). To solve the system of partial differential equations a numerical procedure based on the finite difference method was selected, which includes the spatial and time discretisation, 0.5 mm and 0.25 s, respectively. The finite difference modeling and analysis is conducted using earlier proposed algorithm by the authors [38]. The additional information is given by the measured workpiece temperature T_M , determined at a distance from the grinding wheel/workpiece interface $z=1$ mm (Fig. 6). The grinding input parameters were taken from chapter 4.1. The thermal conductivity is taken as a function of temperature $k=21.378+0.0275 \cdot T$ W/(m°C), the specific cutting force is $k_s=20.378h_m^{-0.728}$ kN/mm² and the energy partition $p(e)=18\%$.

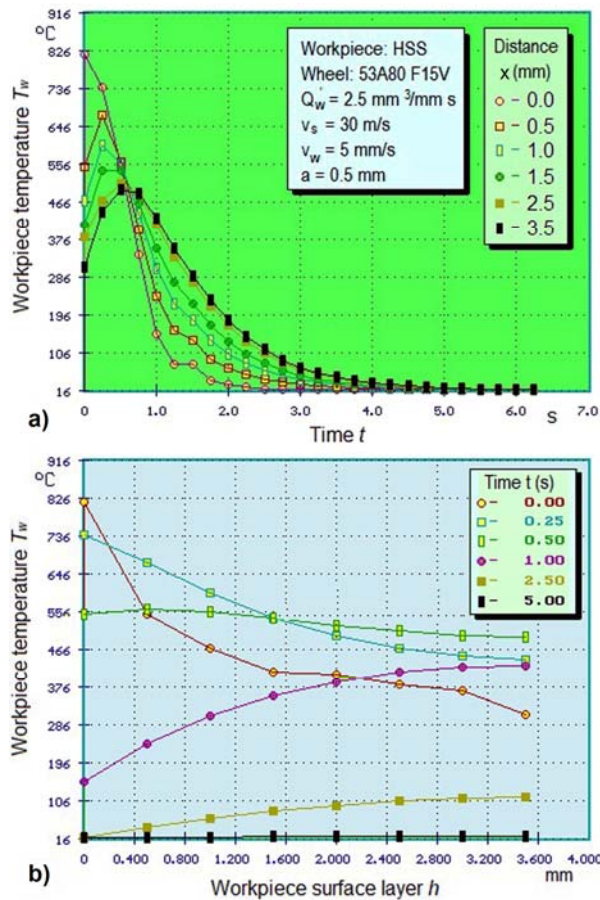


Fig. 9. Modeling of the temperature distribution in the creep-feed grinding: a) Time temperature, b) Workpiece surface layer temperature

For pre-defined conditions, the workpiece temperature profile was obtained, Fig. 9. The inverse model illustrates the temperature change over time at the different distances from the contact surface of the grinding wheel/workpiece interface, that is, the temperature change into the surface layer of the workpiece.

If successive heating of a semi-infinite plates are approximated as a semi-infinite solid, and that the heat source moves at a constant workpiece speed over the workpiece surface, it can be simulated a two-dimensional temperature field in the grinding zone. The temperature distributions in the grinding zone for time periods $t=3, 9$ and 15 s, by the finite element method using ANSYS 16.1 software, are shown in Fig. 10.

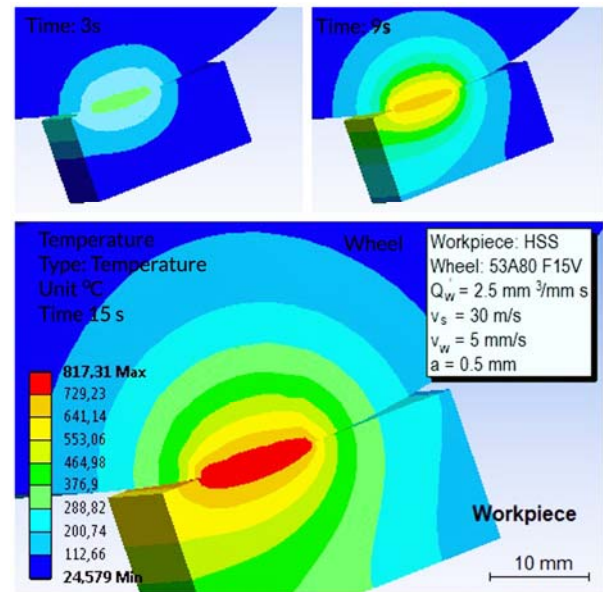


Fig.10. Temperature field of grinding zone

When the grinding temperature distribution is determined, the heat flux over time on the surface layer of the workpiece material is calculated by inverse approach, Fig. 11. The change of the heat flux distribution is determined under the assumption of a minimum heat affected zone (workpiece surface temperature should not be allowed to exceed 550 °C for the selected material).

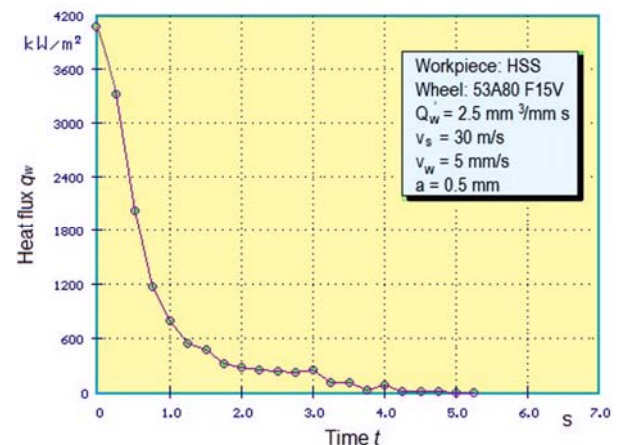


Fig. 11. Heat flux distribution on workpiece

Fig. 12 illustrates two-dimensional distributions of the heat flux in the grinding zone. The heat flux distributions for time periods $t=3, 9$ and 15 s, were determined by the finite element method using ANSYS 16.1 software.

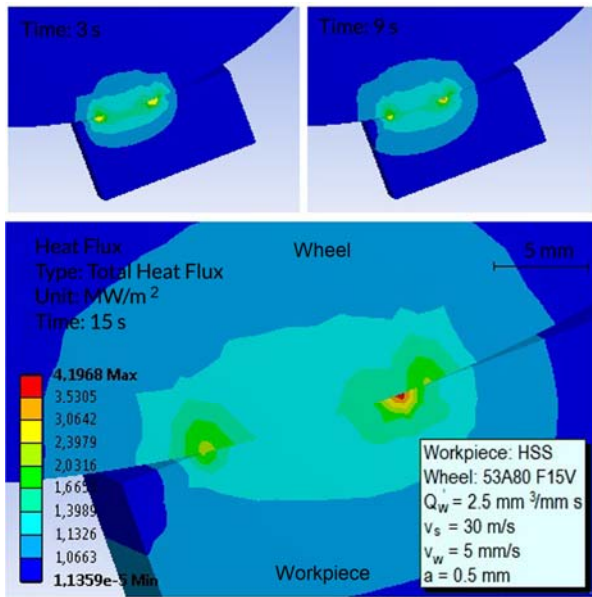


Fig.12. Heat flux distribution in grinding zone

The inverse optimization problem of the creep-feed grinding is carried out under the assumption of a maximum material removal rate ($Q'_w = a \cdot v_w = \max$), Fig. 13. The inverse optimization by the minimization of an objective function U , relies on the determination of the most favourable ratio between the intensity of the heat flux (Eq. 10) and its duration time (Eq. 11). By searching a limited space of the solution, the optimal values of the depth of cut $a=0.126$ mm and the speed of the workpiece $v_w=9.46$ mm/s were determined [38].

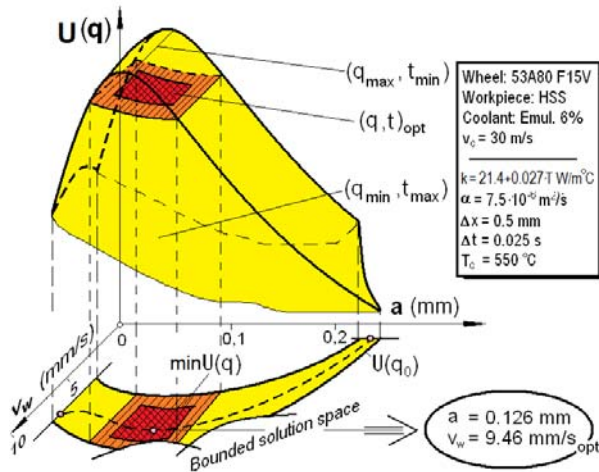


Fig. 13. Inverse optimization model of heat transfer in grinding

5. DISCUSSIONS

The inverse heat transfer problem is a very important optimization technique of the complex machining problems. The inverse optimization method

gives a good insight into the complexity of the machining process, especially since the additional measured values were used. In this context, the inverse optimization problem of the heat transfer is used in the thermal effects of machining that cannot be solved by classical methods.

Compared to other techniques, the inverse optimization problem has a number of advantages. This approach increases the model flexibility and enables a better prediction of thermal performance. The necessary simplification of the model is mitigated with the additional information of an experimental nature. The optimization depends to a lesser extent on model, objective function and boundary conditions conceptualization. The inverse optimization model can be solved using standard optimization methods.

The computed temperature distribution with the inverse method shows a high degree of conformity with the experimentally measured results (mean relative error is less than 5%). High accuracy of the calculation of the workpiece temperature distribution also implies the reliability of the calculated heat flux. In that sense, the inverse optimization problem can be used for prediction of the machining parameters that are suitable for achieving high productivity and quality, without going to the extensive experiments. The comparison between the computed optimal machining performance and derived experimental data, shows slight differences. The differences can be explained by a large number of parameters that had to be estimated during the machining process modeling. In this respect, the inverse optimization problem has proved to be industrially feasible in the case of improvement of the machining process. This approach is especially applicable in the production of advanced products that are difficult to make with machining process.

6. CONCLUSIONS

Based on the available information, advanced machining processes (a good example is creep-feed grinding) are becoming an important segment of the modern manufacturing industry. Advanced machining processes are material removing technologies different from conventional machining processes, and widely used in machining complex surface profile from difficult-to-machine materials. However, the advanced machining processes lead to intense concentration of heat in the cutting zone, which is why special attention is paid to the study of thermal effects.

In recent times, the inverse optimization problem has proved to be very suitable for the thermal analysis of a complex heat transfer in the machining. The inverse optimization problem that is set based on the experimental results provides access that is very close to the real state.

The inverse heat transfer approach enables the identification, simulation and optimization of the temperature and heat flux distribution at the surface of the workpiece during machining. Thereby, the inverse optimization problem is based on locating the most favourable relationship between main heat transfer parameters (heating power and duration). By regulating

the heat transfer parameters, the optimal machining conditions are achieved that enable high productivity and quality of work. The inverse optimization results are quite close to the experimental data.

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