http://doi.org/10.24867/JPE-2022-02-030

Journal of Production Engineering
Vol.25 No.2

JPE (2022) Vol.25 (2) Original Scientific Paper

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THEORETICAL APPROACH ON TWO FACTORIAL DESIGN ON RESIDUAL CURVATURE OF BAR STRAIGHTENING IN CROSS-ROLL ARRANGEMENTS

Received: 01 October 2022 / Accepted: 29 November 2022

Abstract: *Use of straight round bars is necessary in industries due to requirement of raw material as input for production of value-added products. Bars are often available in bent condition; hence straightening is a necessity before commercial use. Therefore, choosing a proper lot of straightened bars with acceptable level of residual curvature is a quality requirement. Use of statistical analysis by Factorial Design will help in decision making through appropriate Test Hypothesis for quality criterion of acceptable range of residual curvatures. This paper has made an attempt to look into the applicability of Factorial Design in bar straightening process.*
Key words: *Roller diameter, straightness, helix angle, residual curvature.*

Teorijsko ispitivanje procesa ispravljanja šipki sa zaostalim krivinama u rasporedu unakrsnih valjaka primenom dvofaktonog dizajn plana. *Upotreba ravnih okruglih šipki je neophodna u industriji zbog potrebe za sirovinom kao inputom za proizvodnju proizvoda sa dodatom vrednošću. Šipke su često dostupne u savijenom stanju; stoga je ispravljanje neophodno pre komercijalne upotrebe. Stoga je izbor odgovarajuće količine ispravljenih šipki sa prihvatljivim nivoom preostale zakrivljenosti uslov kvaliteta. Korišćenje statističke analize faktornog dizajna će pomoći u donošenju odluka kroz odgovarajuće testiranje hipoteza za kriterijum kvaliteta prihvatljivog opsega zaostalih krivina. Ovaj rad je pokušao da ispita primenljivost faktorskog dizajna u procesu ravnanja šipki.*
Ključne reči: *Prečnik valjka, ravnost,, ugao zavojnice, zaostala zakrivljenost.*

1. INTRODUCTION

In general round bars produced in re-rolling industries are often not perfectly straight or it can be stated that straightness is in general compromised after production of round bars. Commercially available round bars are often available with the degree of straightness that varies from 1 in 750 to 1 in 5000 [1]. Bars are therefore usually available with some inbuilt residual curvatures. Sometimes round bars are straightway used with some residual curvatures along the bar length where precision is not a dominant factor. However, everywhere such bars cannot be deployed straightway as built-in residual curvature may not be in acceptable range as far as quality is concerned. Hence, it is quite an essential requirement that bars with residual curvatures are processed with an aim to reduce residual curvature so that the production lot falls in acceptable range.

Actual requirement of straightness of round bars depends on deployment of bars either in raw form or after required straightening through kinematic reverse bending with cross-roll straighteners. However, option remains that in precision application, round bars can be machined to required size as per design requirement. Sometimes machining is not preferred as it would reduce bar diameter. Moreover, if material is costly then machining would result in increase of product cost. However, a bent bar can be made straight through reverse kinematic bending either with the help of cross-roll straighteners or by application of load under locking mechanism [2]. The process of bar straightening in cross-roll arrangement essentially begins with the system of cross-rolls at helix angle, α also termed as roll angle and roller radius, R which correspond to the equations of throughput speed of bar [3,4]. A brief discussion on theoretical aspects of bar straightening will explain the process clearly. Present scope of work is to understand various factors involved in cross-roll straightening and thereupon to look at the significance of experimental design in the process of kinematic reverse bending. Cross-rolls at an angle termed as helix angle is a key parameter in the process. This also causes bars to rotate along the axis and the bar moves forward. Roller diameter plays another important role in the process. Therefore, roller diameter and helix angle may be considered as two factors and can be employed in experimental design. The present paper has been focussed on the statistical aspect of bar straightening process mainly to understand the significance of factors like helix angle and roller diameter in the process of straightening. The analysis of variance will show the F Test value thus enabling to arrive into statistical decision based on the observations.

2. THEORETICAL ASPECTS OF BAR STRAIGHTENING

The round bar's motion in cross-roll arrangement has been described in the schematic diagram (Fig.1a). The cross-rolls are set at helix angle $\pm\alpha$, with respect to bar's axial motion and throughput speed, v_x. The bar rotates about its own axis at an angular velocity $\dot{\theta}$ as shown in Fig.1(b). Fig.1(c) and Fig.1(d) show the diagram of rollers making helix angle α with bar in motion and photograph of rollers and bar or tube in a straightening machine. Radius of bar and rollers are *r* and *R* respectively.

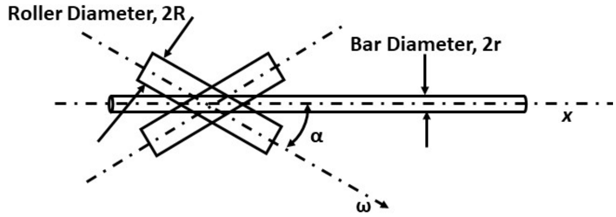

Fig. 1(a). Round bar's motion through Cross-Rolls

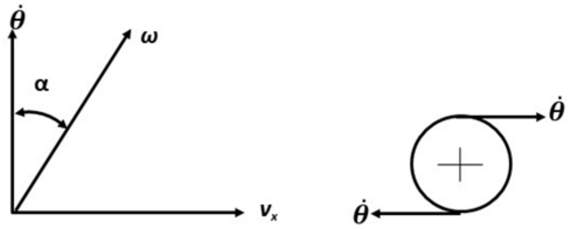

Fig. 1(b). Bar's rotation and angular velocity

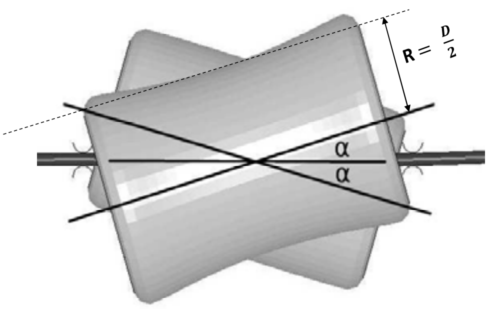

Fig. 1(c). Diagram of roller showing helix angle α [5]

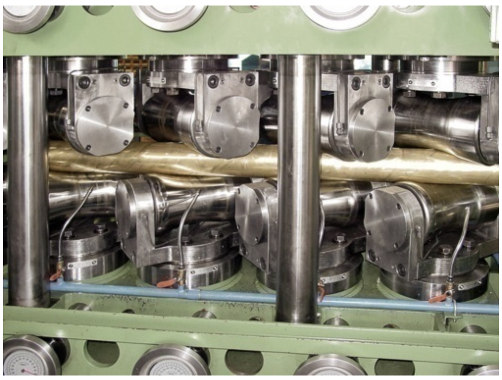

Fig. 1(d). Picture of Cross-Roll in Straightening Machine [6]

Cross-rolls actually cause bars to rotate about its own axis as the bar progresses forward through the rolls. If ω is the angular velocity of the rolls of radius R, throughput velocity $\boldsymbol{v_x}$ and the tangential velocity is $\boldsymbol{v_t}$ of a point on the surface of the bar, then it can be written as [4]

$$v_x = \omega R sin\alpha \quad \text{and} \quad (1)$$

$$v_t = \omega R\ cos\alpha \quad (2)$$

The pitch length of the bar '*p*' can be expressed in terms of helix angle α and bar radius as below

$$p = 2\pi r tan\alpha \quad (3)$$

In non-dimension form,

$$\bar{p} = \frac{p}{l} = 2\pi\bar{r}tan\alpha, \text{ where } \bar{r} = \frac{r}{l} \quad (4)$$

where '*l*' is the length of bar.

3. ANALYSIS OF RESIDUAL CURVATURE IN STRAIGHTENING PROCESSES

Non-dimensional form on the relationship of moment-curvature (*M – C*) is described as below [4]

$$\bar{M} = \bar{C} \qquad 0 \leq \bar{M} \leq 1,$$
$$\bar{M} = \bar{M}(\bar{C}) \qquad 1 \leq \bar{M}, \quad (5)$$

where, $\bar{M} = M/M_y$, and $\bar{C} = C/C_y$

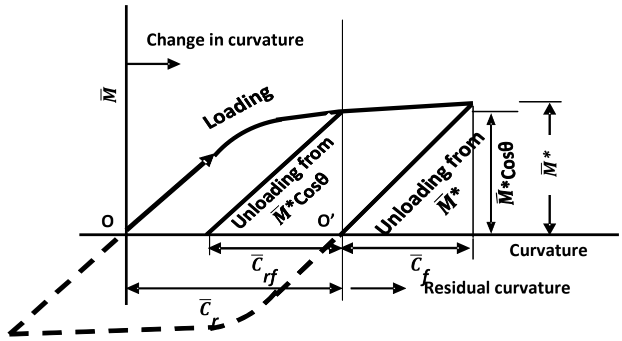

Fig. 2. Relationship between curvature change and resisting moment developed in the bar.

M_y is the yield moment and C_y is the yield curvature for a section considered. $\bar{M}(\bar{C})$ is a function of $\bar{C}$, which depends on the stress-strain relationship of the material beyond the yield point. For convenience purpose, curvature and moment of resistance in reverse loading will be taken as positive. Considering a bar to be straightened is having initial residual curvature, $-\bar{C}_r$, the length of the bar should be subjected to loading in the reverse direction. The loading should be kinematically either by applying a known moment or known curvature. If the loading is kinematic and the final curvature to which the bar length is subjected is $\bar{C}_f$, then, in order to leave no final residual curvature on unloading, the curvature change $\Delta\bar{C}$, as illustrated in Fig. 2 can be written as

$$\Delta\bar{C} = C_f + \bar{C}_r \quad (6)$$

The prime objective of the entire process is to get straight bar practically without any residual curvature or with the range of final residual curvatures acceptable to the user in large scale production so that product lot is within Acceptable Quality Level (AQL).

4. TWO-FACTOR FACTORIAL DESIGN OF FINAL RESIDUAL CURVATURES IN BAR STRAIGHTENING

Based on equations discussed above, it can be seen that ***helix angle*** and ***roller diameter*** play significant roles in the process of bar straightening. Therefore, helix angle and roller diameters may be considered as variables (factors) in factorial design for above process considering one specific bar diameter. However, it is possible to choose bar diameter also as a variable or factor when several bars of various sizes are in consideration. However, at present the focus is on a two-factor factorial design. Through factorial design it may be possible to find a helix angle that is most suitable for the process and can be considered as robust design. As a general case, let final residual curvature C_{fijk} be the observed responses when factor of roller diameter is at ith row (i =1,2,…,a) and factor of helix angle is at jth column (j=1,2,…,b) for kth replication (k=1,2,…,n).

where,

'a' is the level of factor of roller diameter;

'b' is the level of factor of helix angle and

'n' is the number of replications.

In general, a two factorial design for helix angle roller diameter can be described as shown in Table-1.

		Factor of Helix angle for α_j for jth level.			
		α_1	α_2	…	α_b
Factor of Roller Diameter (D_i) for ith level	D_1	$C_{f111}, C_{f112}, \ldots C_{f11n}$	$C_{f121}, C_{f122}, \ldots C_{f12n}$		$C_{f1b1}, C_{f1b2}, \ldots C_{f1bn}$
	D_2	$C_{f211}, C\kappa_{f212}, \ldots C_{f21n}$	$C_{f221}, C_{f222}, \ldots C_{f22n}$		$C_{f2b1}, C_{f2b2}, \ldots C_{f2bn}$
	. . .				
	D_a	$C_{fa11}, C_{fa12}, \ldots C_{fa1n}$	$C_{fa21}, C_{fa22}, \ldots C_{fa2n}$		$C_{fab1}, C_{fab2}, \ldots C_{fabn}$

Table 1. Two Factorial Design for Final Residual Curvatures

The order in which the abn observations are taken is selected at random so that this design is a completely randomized design.

The effects model of the above observations in a factorial experiment can be written as below.

$$C_{fijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \quad \begin{cases} i= 1,2,\ldots,a \\ j= 1,2,\ldots,b \\ k=1,2,\ldots,n \end{cases} \quad (7)$$

where,

μ is the overall mean effect of roller diameter and helix angle,

τ_i is the effect of the ith level of Roller Diameter factor,

β_j is the effect of the jth level of helix angle factor,

$(\tau\beta)_{ij}$ is the effect of interaction between τ_i and β_j, and

ϵ_{ijk} is the random error component.

Both the factors are assumed to be fixed, and the treatment effects are defined as deviations from the overall mean, so that

for Roller Diameter $\sum_{i=1}^{a} \tau_i = 0$ and (8)

for Helix Angle $\sum_{j=1}^{b} \beta_j = 0.$ (9)

Similarly, the interaction effects between roller diameter and helix angle are fixed and defined in following way

$$\sum_{i=1}^{a} (\tau\beta)_{ij} = \sum_{j=1}^{b} (\tau\beta)_{ij} = 0 \quad (10)$$

Considering n number of replication in the experiment, there are 'abn' number of total observations.

It is also possible to express factorial experiment through means model [7] as below.

$$C_{fijk} = \mu_{ij} + \epsilon_{ijk} \quad \begin{cases} i= 1,2,\ldots,a \\ j= 1,2,\ldots,b \\ k= 1,2,\ldots,n \end{cases} \quad (11)$$

where, the mean of the ijth cell is

$$\mu_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} \quad (12)$$

In this case two-variable factorial, both variables are of equal interest. Testing hypotheses about the equality of Roller Diameter treatment effects, can be written as

Null Hypothesis $H_0: \tau_1 = \tau_2 = \ldots = \tau_a = 0$ (13)

Alternate Hypothesis H_1: at least one $\tau_i \neq 0$ (14)

Again, for the equality of helix angle treatment effects, it can be written as

$H_0 : \beta_1 = \beta_2 = \ldots = \beta_b = 0$ (15)

H_1 : at least one $\beta_j \neq 0$ (16)

In case there is interactions of Roller Diameter and Helix angle, then

$H_0 : (\tau\beta)_{ij} = 0$ for all i,j (17)

H_1 : at least one $(\tau\beta)_{ij} \neq 0$ (18)

5. STATISTICAL ANALYSIS OF THE DIAMETER AND HELIX ANGLE FACTORS IN BAR STRAIGHTENING PROCESS

Let $C_{fi..}$ denote the total observations of curvature under the ith level of Diameter factor,

$C_{f.j.}$ denote the total observations of curvature under the jth level of Helix angle factor,

$C_{fij.}$ denote the total of all observations of curvature in the ijth cell, and

$C_{f...}$ denote the grand total of all the observations of curvature.

We define $\bar{C}_{fi..}$, $\bar{C}_{f.j.}$, $\bar{C}_{fij.}$, and $\bar{C}_{f...}$ as the corresponding row, column, cell and grand averages.

Mathematically we can express as below:

$$C_{fi..} = \sum_{j=1}^{b} \sum_{k=1}^{n} C_{fijk},$$

$$\bar{C}_{fi..} = \frac{C_{fi..}}{bn} \qquad i=1,2,..,a \tag{19}$$

$$C_{f.j.} = \sum_{i=1}^{a} \sum_{k=1}^{n} C_{fijk},$$

$$\bar{C}_{f.j.} = \frac{C_{f.j.}}{an} \qquad j=1,2,..,b \tag{20}$$

$$C_{fij.} = \sum_{k=1}^{n} C_{fijk},$$

$$\bar{C}_{fij.} = \frac{C_{fij.}}{n} \qquad i= 1,2,.....,a \quad j= 1,2,.....,b \tag{21}$$

$$C_{f...} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} C_{fijk},$$

$$\bar{C}_{f...} = \frac{C_{f...}}{abn} \tag{22}$$

The total corrected sum of squares may be written as

$$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (C_{fijk}-\bar{C}_{f...})^2$$

$$= \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} [(\bar{C}_{fi..}-\bar{C}_{f...}) + (\bar{C}_{f.j.}-\bar{C}_{f...}) + (\bar{C}_{fij.}-\bar{C}_{fi..}-\bar{C}_{f.j.}+\bar{C}_{f...}) + (C_{fijk}-\bar{C}_{fij.})]^2$$

$$= bn\sum_{i=1}^{a} (\bar{C}_{fi..}-\bar{C}_{f...})^2 + an\sum_{j=1}^{b} (\bar{C}_{f.j.}-\bar{C}_{f...})^2 + n\sum_{i=1}^{a} \sum_{j=1}^{b} (\bar{C}_{fij.}-\bar{C}_{fi..}-\bar{C}_{f.j.}+\bar{C}_{f...})^2 + \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} (C_{fijk}-\bar{C}_{fij.})^2 \tag{23}$$

It can be further expressed the above in a simplified manner as below.

Total sum of squares (SS_T) has been partitioned into following manner

(i) a sum of squares due to factor of Roller Diameter, (SS_{RD});
(ii) a sum of squares due to factor of Helix Angle, (SS_{HA});
(iii) a sum of squares due to interaction between Roller Diameter and Helix Angle (SS_{RDHA});
(iv) a sum of squares due to error (SS_e).

Symbolically, it can be stated as

$$SS_T = SS_{RD} + SS_{HA} + SS_{RDHA} + SS_e$$

The number of degrees of freedom associated with each sum of squares is described in Table 2.

Effect	Degrees of Freedom
Roller Diameter	a-1
Helix Angle	b-1
Roller Diameter & Helix Angle Interaction	(a-1)(b-1)
Error (e)	ab(n-1)
Total	abn-1

Table 2. Degrees of freedom for various factors

Mean Squares (MS) can be evaluated from each sum of squares divided by its degrees of freedom and can be expressed as

$$MS_{RD} = \frac{SS_{RD}}{a-1},$$

$$MS_{HA} = \frac{SS_{HA}}{b-1},$$

$$MS_{RDHA} = \frac{SS_{RDHA}}{(a-1)(b-1)},$$

$$MS_e = \frac{SS_{RD}}{ab(n-1)} \tag{24}$$

It can now be evaluated that the expected values (E) of the mean squares can be expressed by constant variance σ^2 as given below.

$$E(MS_{RD}) = E\left(\frac{SS_{RD}}{a-1}\right) = \sigma^2 + \frac{bn\sum_{i=1}^{a}\tau_i^2}{a-1} \tag{25}$$

$$E(MS_{HA}) = E\left(\frac{SS_{HA}}{b-1}\right) = \sigma^2 + \frac{an\sum_{j=1}^{b}\beta_i^2}{b-1} \tag{26}$$

$$E(MS_{RDHA}) = E\left(\frac{SS_{RDHA}}{(a-1)(b-1)}\right) = \sigma^2 + \frac{n\sum_{i=1}^{a}\sum_{j=1}^{b}(\tau\beta)_{ij}^2}{(a-1)(b-1)} \tag{27}$$

and $$E(MS_e) = E\left(\frac{SS_e}{ab(n-1)}\right) = \sigma^2 \tag{28}$$

If it is assumed that model is adequate and that the error terms ϵ_{ijk} are normally and independently distributed with constant variance σ^2, then each of the ratios of mean squares MS_{RD}/MS_e, MS_{HA}/MS_e, MS_{RDHA}/MS_e can be stated as F distribution with (a-1), (b-1) and (a-1)(b-1) numerators degree of freedom, respectively, and ab(n-1) denominator degrees of freedom, and the critical region would be the upper tail of the F distribution.

The total sum of squares is computed as

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} C_{fijk}^2 - \frac{C_{f...}^2}{abn} \tag{29}$$

The sums of squares for the roller diameter (RD) and helix angle (HA) are

$$SS_{RD} = \frac{1}{bn}\sum_{i=1}^{a} C_{fi..}^2 - \frac{C_{f...}^2}{abn} \tag{30}$$

$$SS_{HA} = \frac{1}{an}\sum_{j=1}^{b} C_{f.j.}^2 - \frac{C_{f...}^2}{abn} \tag{31}$$

For roller diameter and helix angle interaction, SS_{RDHA} may be evaluated in two stages. First step is the computation on the sum of squares between the ab cell totals, which may be termed as sum of squares due to "sub-totals"

$$SS_{Subtotals} = \frac{1}{n}\sum_{i=1}^{a} \sum_{j=1}^{b} C_{fij.}^2 - \frac{C_{f...}^2}{abn} \tag{32}$$

The sum of squares also contains SS_{RD} and SS_{HA}.

Therefore, the second step is to compute SS_{RDHA} as

$$SS_{RDHA} = SS_{Subtotals} - SS_{RD} - SS_{HA} \tag{33}$$

It may be computed SS_E by subtraction as

$$SS_e = SS_T - SS_{RDHA} - SS_{RD} - SS_{HA} \tag{34}$$

6. ANALYSIS OF VARIANCE (ANOVA) FOR FINAL RESIDUAL CURVATURE.

The final residual curvature has been analysed by using ANOVA based on the variables of roller diameter (RD) and helix angle (HA) in the cross-roll bar straightening process. The usual procedure is to employ a statistical software package to conduct an ANOVA. However, mathematical computing of the sums of squares is necessary for developing the software. Analysis of variance (ANOVA) for the two variables RD, HA and its corresponding interaction factor (RDHA) along with its error(e) component is described in Table-3.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Roller Diameter (*RD*) Treatments	SS_{RD}	a-1	$MS_{RD} = \frac{SS_{RD}}{a-1}$	$F_0 = \frac{MS_{RD}}{MS_e}$
Helix Angle (*HA*) Treatments	SS_{HA}	b-1	$MS_{HA} = \frac{SS_{HA}}{b-1}$	$F_0 = \frac{MS_{HA}}{MS_e}$
Interactions between Roller Diameter & Helix Angle (*RDHA*)	SS_{RDHA}	(a-1)(b-1)	$MS_{RDH} = \frac{SS_{RDHA}}{(a-1)(b-1)}$	$F_0 = \frac{MS_{RDHA}}{MS_e}$
Error (e)	SS_e	$ab(n$-1)	$MS_e = \frac{SS_{RD}}{ab(n-1)}$	
Total	SS_T	*abn-1*		

Table 3. Analysis of Variance for the variables of roller diameter and helix angle

7. DISCUSSION

The theoretical aspects of bar straightening have been briefly discussed with prime consideration of using statistical theories considering two important process variables i.e. roller diameter and helix angle in the bar straightening process. It is pertinent to consider that roller diameter and helix angle of rollers play significant roles in the process which is easily understood from the various equations. Considering variables like Roller diameter and Helix angle as factors is essential so as to understand the significance of the observations of final curvatures based on above under different values of both factors. Statistical aspects have been reasonably detailed like sum of squares and means thereof have been evaluated based on curvature values. A systematic approach has been taken to arrive into mean square values based on factors and their interactions along with error term. The degrees of freedom for both the factors have been taken care of based on their levels.

8. CONCLUSION

Application of ANOVA based on factors of helix angle and roller diameter has become now quite simple. The above process shall hold good by *F* Test and compare the value from the *F* Table so as to arrive into statistical conclusion for an appropriate hypothesis. The possibility of inclusion of other factors like type of material i.e. elastic/plastic modulus and bar diameter remains. Considering more factors for ANOVA will make the study more elaborate. This type of investigation may develop useful tool for justification of role and importance on various factors in the process of bar straightening which can certainly form the basis for process improvement.

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