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Bodzás, S.

FINITE ELEMENT METHOD ANALYSIS OF VARIOUS TOOTH ROOTS ON THE PINION OF CONNECTING HELICAL GEAR PAIRS HAVING COMPLEX TEETH BY CONSTANT MOMENT

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Abstract: *The aim of this study is the analysis of the mechanical parameters on the tooth roots having different fillet radiuses in case of helical gears having modified teeth. These gear pairs are widely used in different engineering constructions because of the better load transmission and the more silent noise. Consequently, the development researches are determinable in this field. Designing of one helical gear is needed for the analysis. After that the creation of its CAD model is essential. This gear pair is analyzed by different fillet radiuses on the tooth root of the pinion. All of the other geometric parameters are unchanged. The applied moments are also permanent for each experiment. After the FEM analysis the received results and graph will be evaluated.*
Keywords: *helical gear, fillet, tooth root, stress, deformation*

FEM analiza različitih korena zuba na zupčaniku kod spojnih parova zupčanika sa složenim zupcima uz konstantni moment. *Cilj ovog rada je analiza mehaničkih parametara na korenima zuba različitih poluprečnika zavojnice u slučaju spiralnih zupčanika sa modifikovanim zupcima. Ovi parovi zupčanika se široko koriste u različitim inženjerskim konstrukcijama zbog boljeg prenosa opterećenja i nižeg nivoa buke. Shodno tome, razvojna istraživanja su izvršena u ovoj oblasti. Za analizu je potrebno projektovanje jednog spiralnog zupčanika. Nakon toga je od suštinskog značaja kreiranje njegovog CAD modela. Ovaj par zupčanika se analizira pomoću različitih poluprečnika zavojnice na korenu zuba zupčanika. Svi ostali geometrijski parametri su nepromenjeni. Primenjeni momenti su takođe trajni za svaki eksperiment. Nakon FEM analize dobijeni rezultati i grafikon će biti evaluirani.*
Ključne reči: *spiralni zupčanik, ugao, koren zuba, naprezanje, deformacija*

1. INTRODUCTION

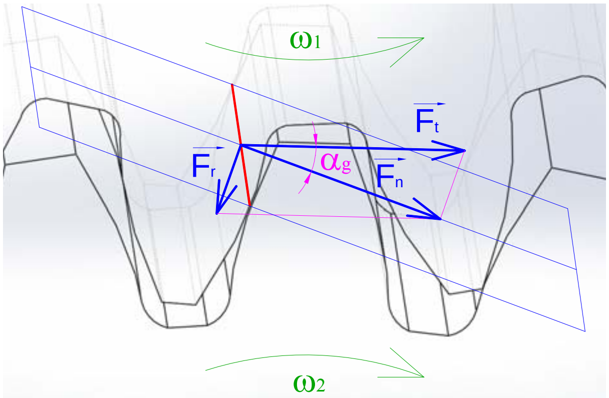
The spur gear's teeth can take up the pressure force at the same time along the tooth length. After the connection, this pressure force is ceased at the same time. This load changing and the suddenly direction change of the friction force on the main point cause dynamical effects and vibrations especially in case of higher circumferential velocity (Figure 1.a) [5, 6].

The tooth connection is continuous in case of helical gears because of the skew tooth direction. That is why the working is calmer and more noiseless (Figure 1.b). The disadvantage is the axial force which could be compensated by Herringbone gear [5, 6].

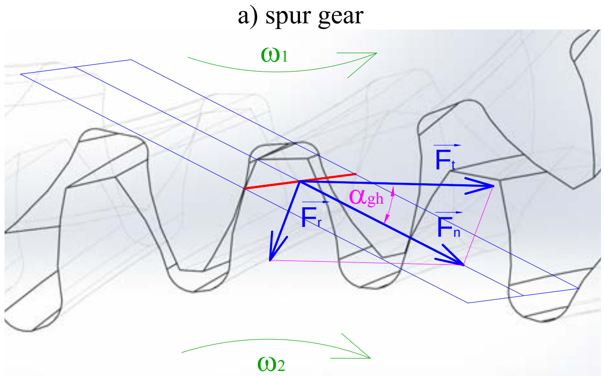
The main property of the helical gear having modified teeth is the sum of the two addendum modification coefficients are zero [1, 3, 5, 6, 7]:

$$x_{1h} + x_{2h} = 0 \rightarrow x_{1h} = -x_{2h} \qquad (1)$$

In spite of this the tooth connection is happened on the pitch circle diameter (Figure 2) [1, 3, 5, 6, 7]. Knowing of the reference's recommendations [3-7] a Matlab computer software was developed to the easement of the designing process. Knowing of the input parameters this software can calculate all of the geometric parameters and the involute profile points [1, 2]. The calculated starting parameters could be seen on Table 1 [2]. The CAD models could be seen on Figure 4.a.



a) spur gear



b) helical gear

Fig. 1. Tooth connections and the position of the load force components

The helical gear could be manufactured by the same conventional machines (Sunderland, Fellows and Pfauter) than the spur gear [5, 6, 7].

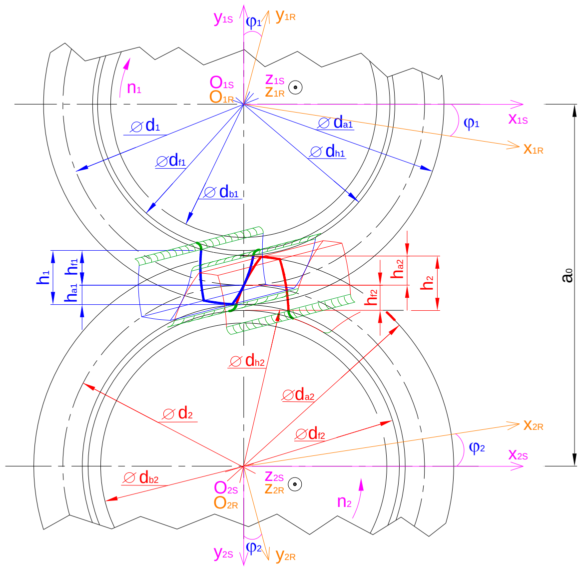
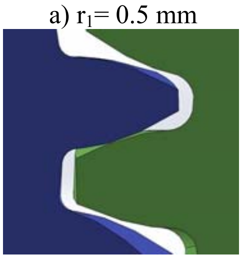


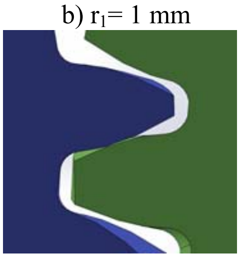
Fig. 2. The geometric parameters of the helical gear having modified teeth (the fillet radiuses are green)

a) r_1= 0.5 mm

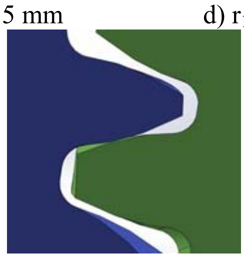
b) r_1= 1 mm



c) r_1= 1.5 mm



d) r_1= 2 mm



e) r_1= 2.5 mm

Fig. 3. The application of different fillet radiuses on the tooth root of the pinion (blue)

Knowing of the involute curves of the tooth gears the connection points could be determined by mathematical way using of the coordinate system arrangement on Figure 2 [4]. The contact points of the gear pairs could be determined by the following equation:

$$\vec{n}_{1R} \cdot \vec{v}_{1R} = \vec{n}_{2R} \cdot \vec{v}_{2R} = 0 \qquad (2)$$

The main geometric parameters	***Gear drive I.***	***Gear drive II.***	***Gear drive III.***	***Gear drive IV.***	***Gear drive V.***
Fillet radius on the pinion (r_1) [mm]	0.5	1	1.5	2	2.5
Fillet radius on the driven gear (r_2) [mm]	2.6				
Axial module (m) [mm]	5				
Number of teeth of the pinion (z_1)	22				
Number of teeth of the driven gear (z_2)	30				
Base profile angle (α_0) [°]	20				
Tooth trace (β_0) [°]	15				

Table 1. The input parameters and the different fillet radiuses on the pinion

The tooth root is situated between the limit circle (d_h) and the root circle (d_f) (Figure 2) [5, 6]. The modifications of the fillet radius on the pinion could be seen on Figure 3.
The transformation matrix between the elements (Figure 2):

$$M_{2R,1R} = \begin{bmatrix} \cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2 & -\cos\varphi_1 \cdot \sin\varphi_2 - \sin\varphi_1 \cdot \cos\varphi_2 & 0 & a \cdot \sin\varphi_1 \\ -\sin\varphi_1 \cdot \cos\varphi_2 - \cos\varphi_1 \cdot \sin\varphi_2 & \sin\varphi_1 \cdot \sin\varphi_2 - \cos\varphi_1 \cdot \cos\varphi_2 & 0 & a \cdot \cos\varphi_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

The aim of the Tooth Contact Analysis (TCA) and Finite Element Method Analysis (FEM) [8] in the gear researches [1-2, 9-14] is the determination of the mechanical parameters (deformation, stress, etc.) by different loads (moment or force) to improve the gear geometry for given utilizations.

2. METHODS AND MATERIALS

2.1. The adoption of the coordinate systems

Four coordinate systems are needed for the *FEM* analysis:

- two coordinate systems to the middle points of the connecting gear elements,
- two coordinate systems to the fillets of the tooth roots on the pinion: the *'x'* axes are perpendicular to the fillet surfaces (normal direction).

2.2. Generation of the FEM mesh

The 0.2 mm element size was used on environments of the tooth roots of the pinion. Automatic meshing was applied on the outsides areas (Figure 4.b). The shape of the meshing was tetrahedron. All of the freedom degrees of the driven gear were fixed. Five freedom degrees were fixed on the pinion. Only the rotation around the axis of rotation was allowed. This pinion was loaded by 700, 800 and 900 Nm moments. The selected material type was structural steel [1].

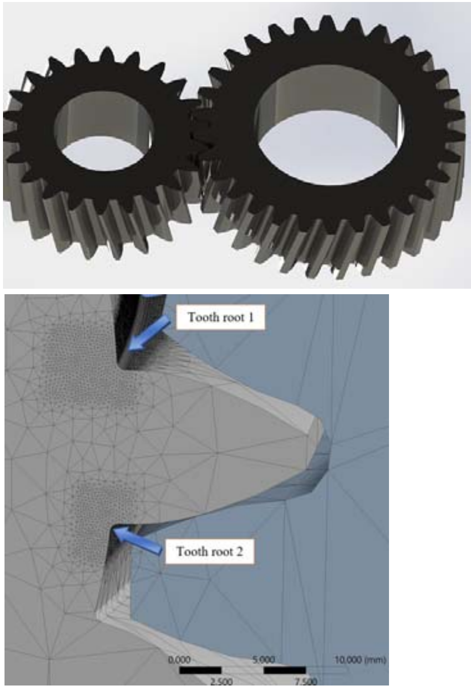
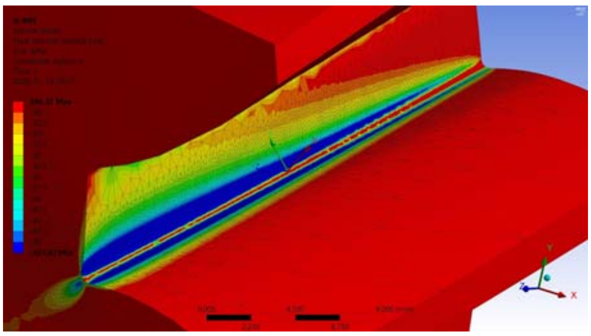


Fig. 4. The CAD models of the helical gears (a) adoption of the FEM mesh (b)

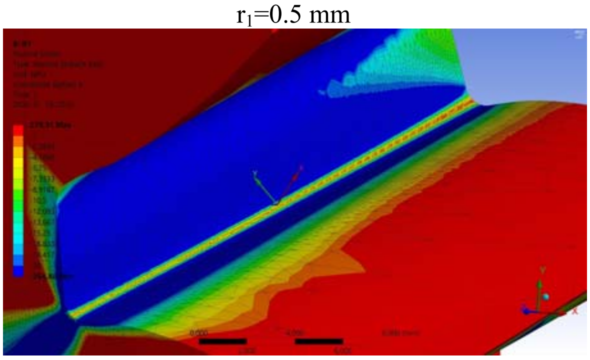
3. RESULTS AND DISCUSSION

3.1. Normal stress analysis

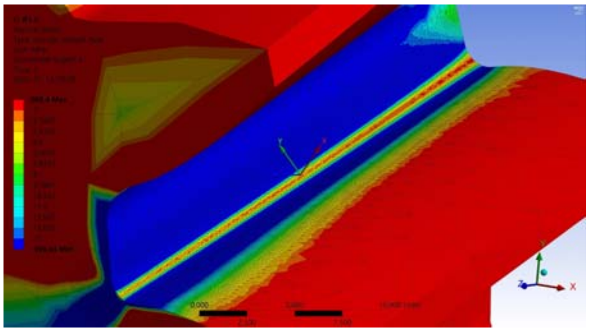
This analysis was created for 700, 800 and 900 Nm moments. The received normal stress results for both tooth roots of the pinion on the contact teeth by 800 Nm load moment could be seen on Figure 5. The same scales were used on each subfigure.



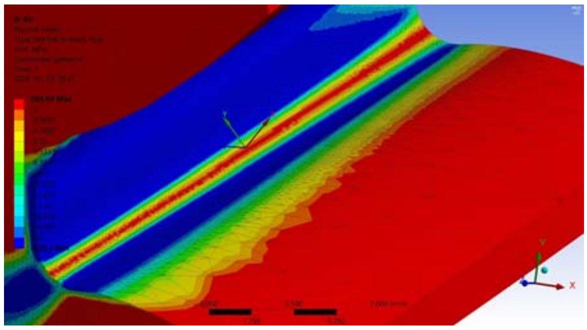
r_1=0.5 mm



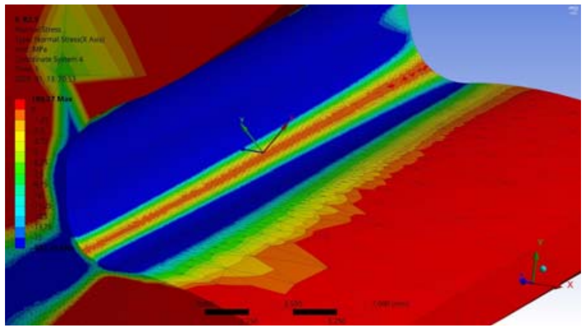
r_1=1 mm



r_1=1.5 mm

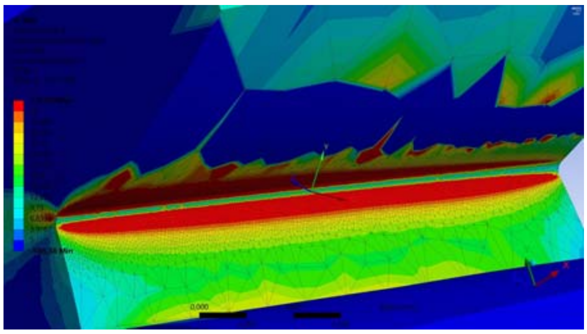


r_1=2 mm

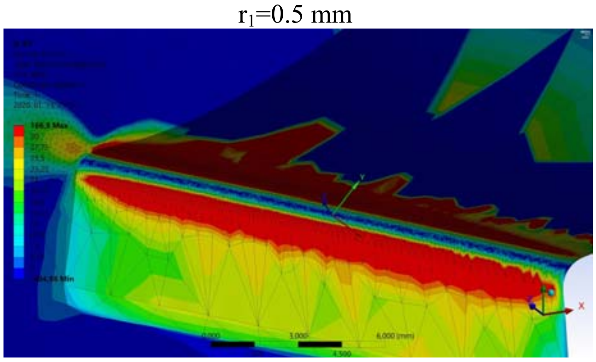


r_1=2.5 mm

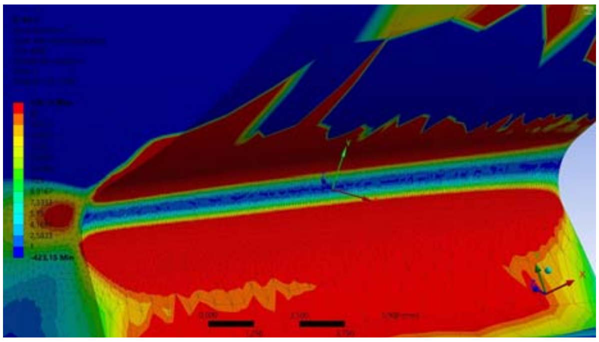
a) 1st tooth root



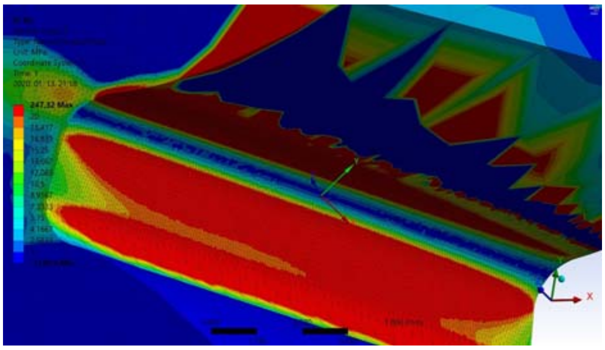
r_1=0.5 mm



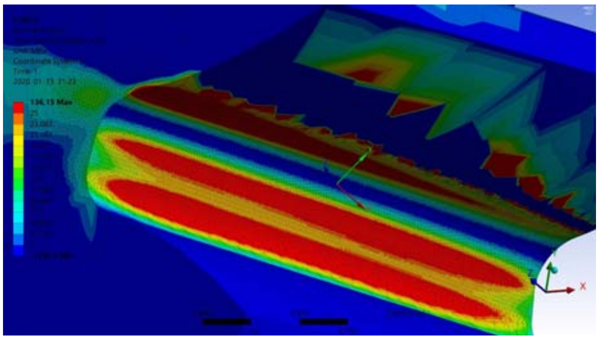
r_1=1 mm



r_1=1.5 mm



r_1=2 mm

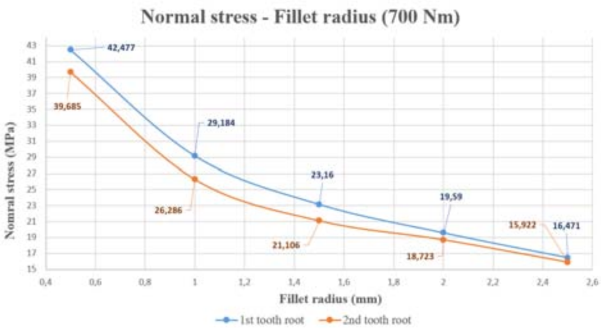


r_1=2.5 mm

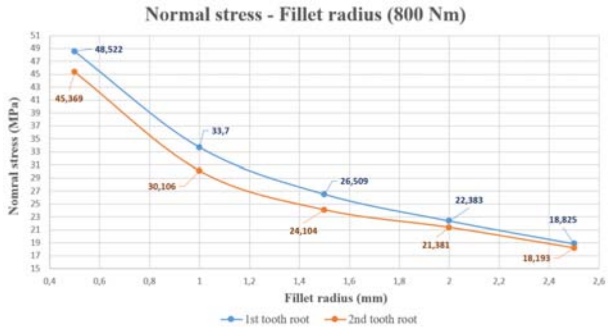
b) 2nd tooth root

Fig. 5. The distributions of the normal stress on the tooth roots of the pinion (800 Nm)

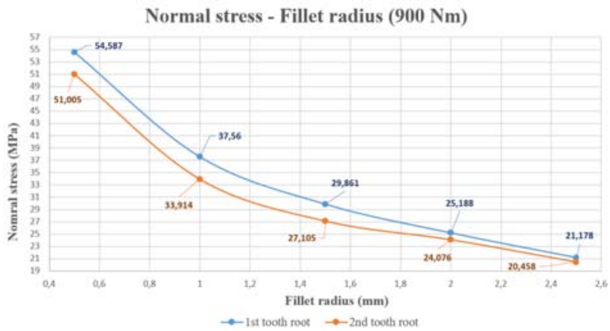
Based on Figure 6 hyperbola shapes were received that is the mathematical function of the inverse proportion. We receive the same graph's shapes for different moments. It means the lower the fillet radius is on the tooth root of the pinion the higher the average normal stress is beside of the constancy of fillet radius on the tooth root of the driven gear. The normal stress results are higher on the 1st tooth root than on the 2nd tooth root. Based on these results the application of the r_1=2.5 mm fillet radius is favorable.



a) M=700 Nm



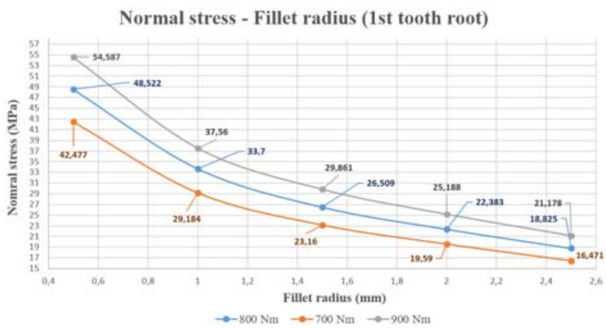
b) M=800 Nm



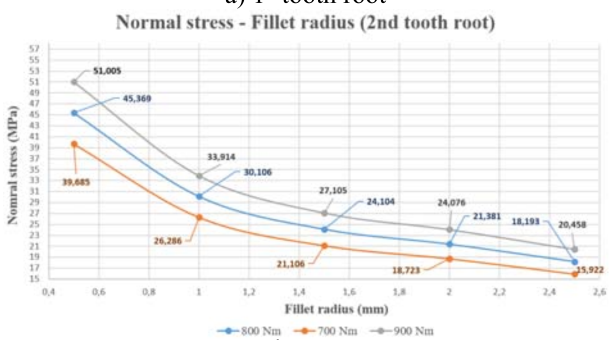
c) 900 Nm

Fig. 6. The function between the fillet radiuses and the average normal stresses on each moment

The normal stress results could be seen on Figure 7 for the two analyzed tooth roots for different moments. The more the moment, the more the normal stress on the tooth roots.



a) 1st tooth root

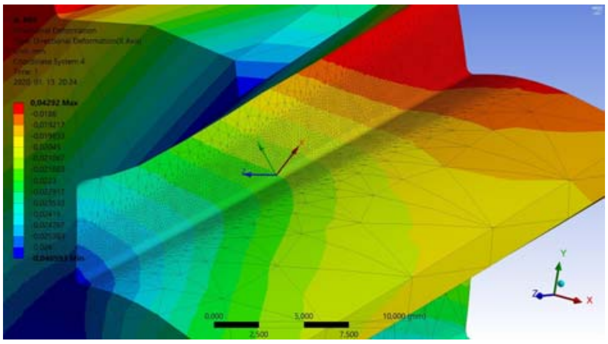


b) 2nd tooth root

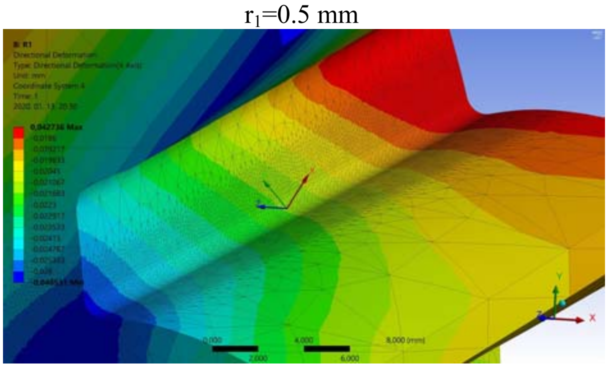
Fig. 7. The normal stress results on both tooth roots

3.2. Normal deformation analysis

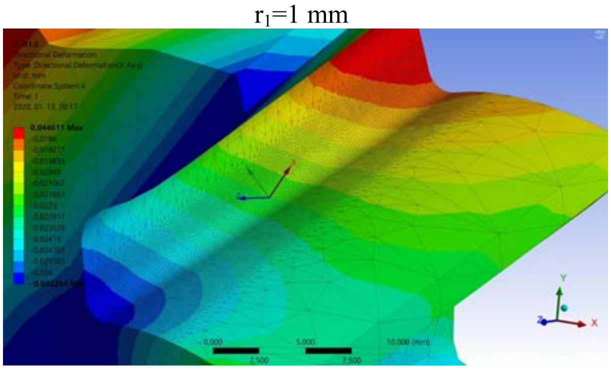
This analysis was created for 700, 800 and 900 Nm moments. The received normal deformation results for both tooth roots of the pinion on the contact teeth by 800 Nm load moment could be seen on Figure 8. The same scales were used on each subfigure.



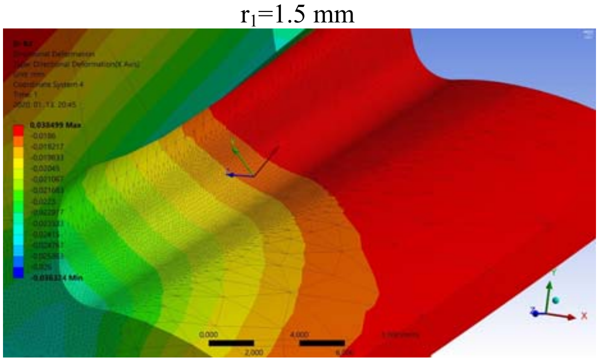
r_1=0.5 mm



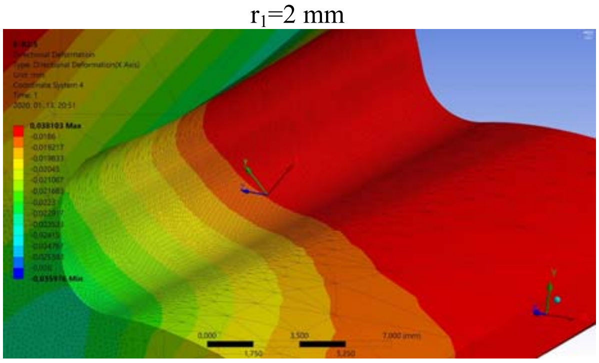
r_1=1 mm



r_1=1.5 mm

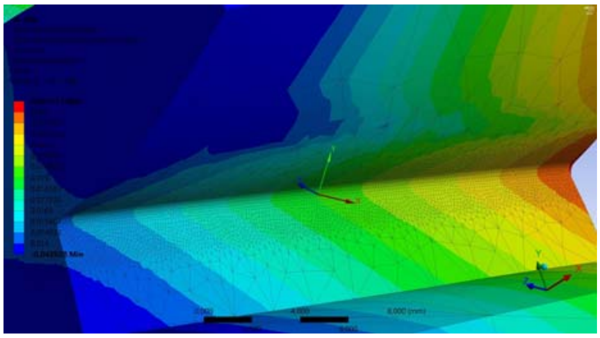


r_1=2 mm

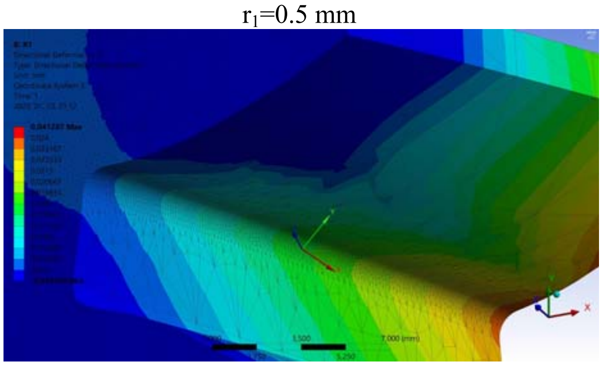


r_1=2.5 mm

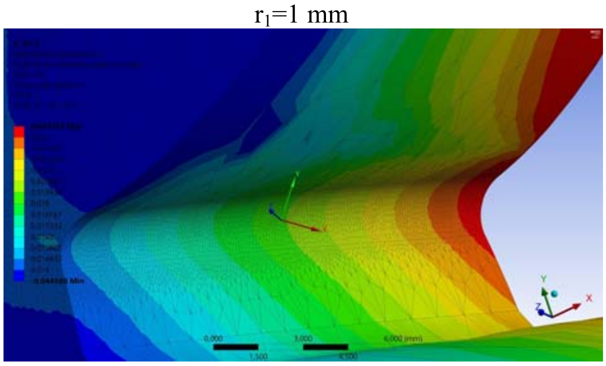
a) 1st tooth root



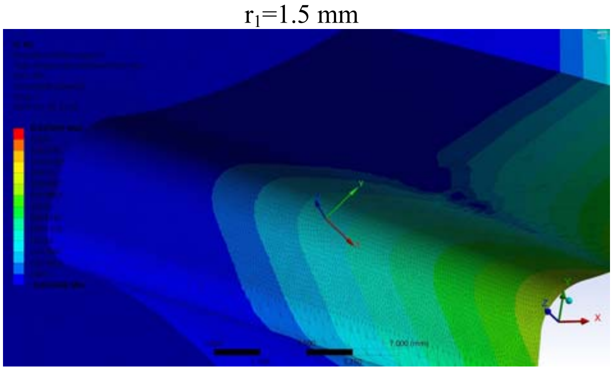
r_1=0.5 mm



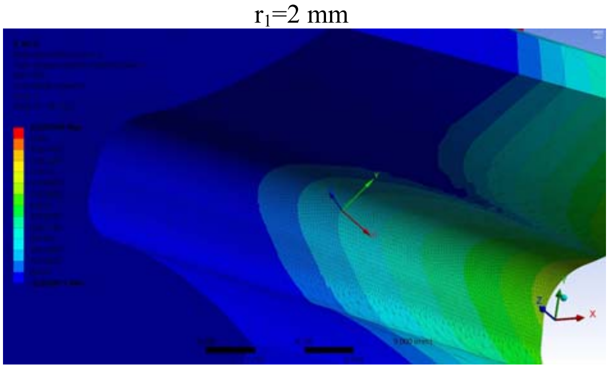
r_1=1 mm



r_1=1.5 mm



r_1=2 mm

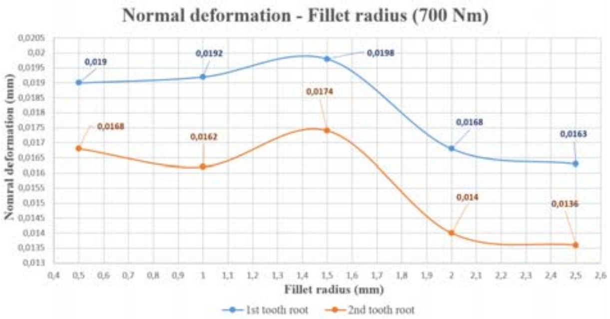


r_1=2.5 mm

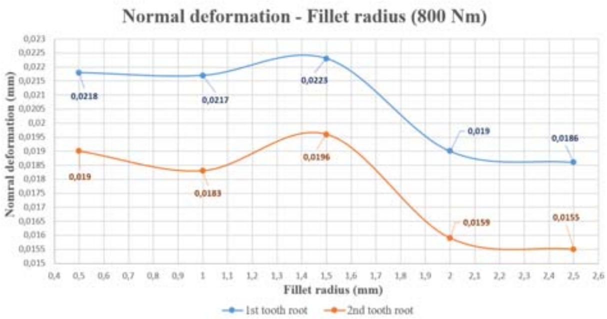
b) 2nd tooth root

Fig. 8. The distributions of the normal deformation on the tooth roots of the pinion (800 Nm)

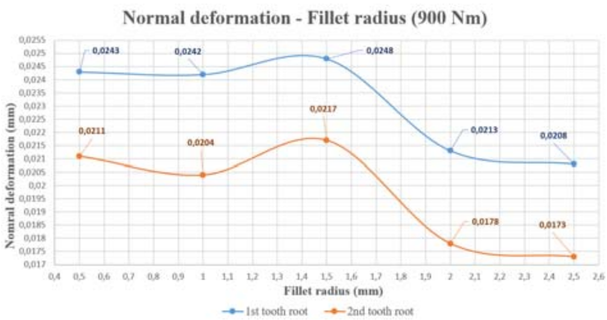
The graph's shapes are the same in case of different load moments (Figure 9). It is obvious the higher the load moment is the higher the normal deformation is. The highest peaks are received approximately in case of r_1=1.5 mm radius and its environment. The lowest deformations are on the values of the higher radius. According to our analysis, the favorable radius is r_1=2.5 mm.



a) M=700 Nm

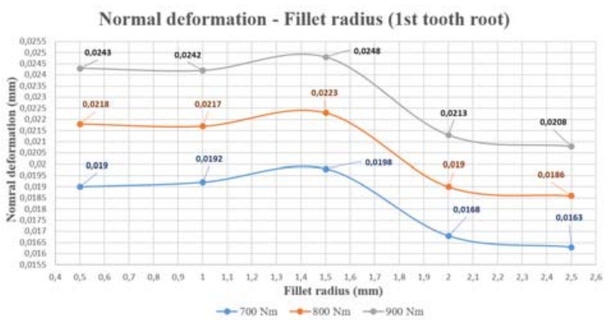


b) M=800 Nm

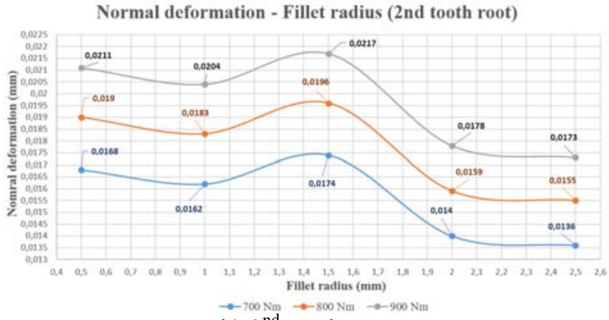


c) M=900 Nm

Fig. 9. The function between the fillet radiuses and the average normal deformations on each moment



a) 1st tooth root



b) 2nd tooth root

Fig. 10. The normal deformation results on both tooth roots

The normal deformation results could be seen on Figure 10 for the two analyzed tooth roots for different moments. The more the moment, the more the normal deformation on the tooth roots. The shape of the function is wavering. The highest normal deformations are in case of r_1=1.5 mm on each tooth root.

3. CONCLUSIONS

The application of helical gears is important to different gear boxes especially in vehicle gear boxes. Consequently, the development researches of these gear pairs are essential to guarantee appropriate geometry for the achievement of good load transmission and working life.

One helical gear drive was designed to analysis the mechanical properties (normal stress, normal deformation) on the tooth roots of the pinion by the modification of the fillet radiuses beside of the constancy of the other geometric parameters. It meant five pinion having different fillet radiuses. The construction designing was created by my-developed Matlab software based on the references. After that the CAD models and the geometrical correct assembly could be done. The applied load moments were the same for each pinion on the FEM analysis. The experiment was repeated three times for three different moments.

As a result, we received similar graph's shapes for the normal stress and the normal deformation results. The lower the fillet radius is on the tooth root of the pinion the higher the average normal stress is beside of the constancy of fillet radius on the tooth root of the driven gear. The normal stress results are higher on the 1st tooth root than on the 2nd tooth root. Based on these results the application of the r1=2.5 mm fillet radius is favorable. The shape of the function is hyperbola, which is the shape of the inverse proportion.

Analysing the normal deformation the highest peaks are received approximately in case of r1=1.5 mm radius and its environment. The lowest deformations are on the values of the higher radius. According to our analysis, the favorable radius is r1=2.5 mm. The shape of the function is wavering.

The connecting points from tooth to tooth could be determinable accordingly Figure 2 knowing the mathematical transformations, the connection equation and the equation of the involute curves.

4. NOMENCLATURE

Symbol	Name	Unit
ω_1, ω_2	angular velocities	1/s
α_g	pressure angle	°
α_{gh}	pressure angle on front section	°
α_0	base profile angle	°
F_n	pressure force	N
F_r	radial pressure force component	N
F_t	axial pressure force component	N
n_1, n_2	number of revolutions	1/s
x_{1h}, x_{2h}	addendum modification coefficients	
O_{1S}, O_{1R}, O_{2S}, O_{2R}	origins of the appropriate coordinate systems	
x, y, z	perpendicular directions of the appropriate coordinate systems	
d_1, d_2	pitch circle diameters	mm
d_{f1}, d_{f2}	dedendum circle diameters	mm
d_{a1}, d_{a2}	addendum circle diameters	mm
d_{b1}, d_{b2}	base circle diameters	mm
d_{h1}, d_{h2}	clearance circle diameters	mm
h_1, h_2	whole depth	mm
h_{a1}, h_{a2}	addendum	mm
h_{f1}, h_{f2}	dedendum	mm
φ_1, φ_2	angular displacement	mm
a_0	elementary centre distance	mm
r_1, r_2	fillet radiuses	mm
m	axial module	mm
β_0	tooth trace	°
z_1, z_2	number of teeth	
n_{1R}, n_{2R}	normal vectors of the connecting surfaces	
v_{1R}, v_{2R}	rotation velocities	mm/s
$M_{2R,1R}$	transformation matrix between the elements	

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Authors: **Dr. Sándor Bodzás, Ph.D.,** Associate professor, Deputy head of department, Department of Mechanical Engineering, University of Debrecen, Debrecen, Ótemető str. 2-4, 4028, Hungary
e-mail: bodzassandor@eng.unideb.hu