http://doi.org/10.24867/JPE-2018-02-060

Journal of Production Engineering
Vol.21 No.2

**JPE (2018) Vol.21 (2)** **Original Scientific Paper**

**Tiwari, I., Laksha, Khanna, P.**

# DEVELOPMENT OF A MATHEMATICAL MODEL TO PREDICT THE PERFORMANCE OF A VIBRATORY BOWL FEEDER FOR HEADED COMPONENTS

Received: 11 July 2018 / Accepted: 20 October 2018

***Abstract:*** *Technology has helped to change the entire world. And in manufacturing industries, automation has become the competitive advantage. Automation helped to produce mass products at desired speed and rate with repeatability. It saves time and manual labour. Mechanised feeding has emerged as critical operation as to ensure discrete part feeding to mass producing assembly lines in a desired orientation and at a desired rate, eliminating the need of human operator for the job. The objective of our investigation is to study the feed behaviour of the headed components such as nails by a vibratory bowl feeder. Looking at the asymmetry of shape of these components slight modifications were made in the basic machine to facilitate their feeding. The response parameter in this case is feed rate and the important input parameters were part size, part population and frequency of vibrations. The experiments were conducted using the methodology of design of experiments. Central composite face cantered technique was used for developing the mathematical model. The adequacy of this model was tested by ANOVA analysis and significance of the regression coefficient was checked and the model was optimised by using RSM.*

***Key words:*** *Vibratory Bowl Feeder, mathematical modelling, ANOVA, optimisation, Response Surface Methodology*

**Razvoj matematičkog modela za određivanje glavnih komponenti vibracionog uređaja za doziranje.** *Tehnologija je pomogla da se promeni ceo svet. A u prerađivačkoj industriji, automatizacija je postala konkurentna prednost. Automatizacija je pomogla u proizvodnji masovnih proizvoda pri željenoj brzini i stopi ponavljanja. To štedi vreme i ručni rad. Mehanizovano doziranje se pojavilo kao kritična operacija kako bi se osiguralo da se diskretni deo dovodi do masovnih produkcijskih linija u željenoj orijentaciji i po željenoj brzini, eliminišući potrebu za ljudskim radom. Cilj naše istrage je proučavanje ponašanja dozatora sa zaglavljenim komponentama kao što su ekseri pomoću vibracionog mehanizma za doziranje. Gledajući asimetriju oblika ovih komponenti, u osnovnoj mašini su napravljene neznatne modifikacije kako bi se olakšalo njihovo doziranje. Izlazni parametar u ovom slučaju je brzina doziranja i važni ulazni parametri su veličina dela, deo populacije i učestalost vibracija. Eksperimenti su sprovedeni metodom plana eksperimenata. Za razvijanje matematičkog modela korišćen je centralni kompozicioni plan. Adekvatnost ovog modela testirana je ANOVA analizom i proveren je značaj koeficijenta regresije, a model je optimizovan korišćenjem RSM-a.*

***Ključne reči:*** *Vibraciona posuda za doziranje, matematičko modeliranje, ANOVA, optimizacija, metod odzivne površine*

## 1. INTRODUCTION

Concept of feeding has gained a significant importance in previous few years because of its usefulness in serving the assembly lines. The pioneering work in the field was carried decades ago by Dr. Geoffrey Boothroyd [1], who in his elaborated work on different types of feeding system has diverted the attention towards the importance of mechanised feeding. A variety of feeding systems available in the industry to suit different requirements. To name a few are bowl feeder, centrifugal feeder, step feeder, linear feeder, etc. Keeping in view the capability of vibratory bowl feeder to handle different shapes and sizes of industrial components, the same was selected for the study. To further substantiate the importance of this feeder a literature survey was carried out and the findings of different experimenters was observed. . Jindal et. al. [2] carried out investigation work on this feeder for clip shaped components and concluded that feed rate increases with increase in frequency and part population. Pandey et. al. [3, 4] investigated the performance of vibratory bowl feeder for threaded fasteners and concluded that feed rate increases significantly with frequency of vibration but decreases with increase in part population. Maher et. al. [5] further carried out investigation with the objective to contribute to the development of a flexible vibratory bowl feeder.

Headed components of a vast variety are extensively used in assembly operations for fastening and joining. Because of their asymmetric shape they pose a challenge to feed them in the desired orientation in an assembly line. Present work, therefore is an attempt to address this problem by conducting a series of experiments in a structured way to establish a relationship between the important process parameters and the response which is feed rate in this case. It is assumed that the developed model will not only help analyse the performance but also shall be able to predict its behaviour at different combinations of process parameters. A popular approach of design of experimentation was used to decide the number of experiments to be conducted so that direct and interaction effects of parameters can be investigated on the feeder output. Central composite face centred technique was used to develop the model followed by application of Response Surface Methodology to optimise the model.

## 2. EXPERIMENTAL SET UP

The experimental set up used for the present investigation is shown in figure 1 which consists of a vibratory bowl feeder with bowl size 300mm. The system is provided with a separate control unit with a provision to vary the frequency of vibrations. As the subject components are headed fasteners, it becomes difficult to ensure their smooth flow and a particular orientation delivery owing to their asymmetrical shape. Slight modifications were made to ensure one component at a time making to the delivery point moreover a special delivery chute was designed and attached at the delivery point to ensure each component aligns itself in its “heads up” position irrespective of the kind of orientation it was in before reaching the delivery chute.

Fig. 1. The experimental set up

## 3. PLAN OF INVESTIGATION

The following steps were considered while carrying out the present investigation.

- Identification of important process parameters and finding and their working ranges.
- Development of a design matrix
- Conducting the experiments as per the design matrix.
- Developing the mathematical model
- Checking the significance of the developed model
- Results and their discussions
- Conclusions

### 3.1 Identification of important process parameters and finding and their working ranges.

A series of trial experiments were conducted to identify the independently controllable process parameters and their working limits. The important process parameters significantly affecting the performance of feeder were found to be the frequency of vibration, part size and part population. Their working ranges were decided on the basis of the intervals in which feeder was found to give stable outputs without any erratic observations. The upper limit is shown as (+1), lower limit as (-1) and the intermediate value as (0). The working limits so decided are given in Table 1.

| No. | FREQUENCY (Hz) | PART SIZE (mm) | PART POPULATION |
|---|---|---|---|
| 1 | 40 (-1) | 25 (-1) | 150 (-1) |
| 2 | 42 (0) | 38 (0) | 200 (0) |
| 3 | 45 (+1) | 51 (+1) | 250 (+1) |

Table.1 The input parameters and their working ranges

### 3.2 Development of a design matrix

The total number of experiments to be conducted were determined by using design expert software. In the present case central composite face centred technique was used with each parameter varying at three levels. The total number of experiments came out to be 20 { $2^3$ (full factorial points) + 2*3(star points) + 6(centre points) = 20 }. All the possible combinations as per the present approach are given in the designed matrix shown in Table 2.

| Std | Run | Factor 1 A: FREQUENCY Hertz | Factor 2 B: Part Size mm | Factor 3 C: Part Population | Response 1 Feed Rate parts/min |
|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | -1 | 55 |
| 18 | 2 | 0 | 0 | 0 | 26 |
| 4 | 3 | 1 | 1 | -1 | 41 |
| 8 | 4 | 1 | 1 | 1 | 58 |
| 13 | 5 | 0 | 0 | -1 | 22 |
| 19 | 6 | 0 | 0 | 0 | 23 |
| 20 | 7 | 0 | 0 | 0 | 18 |
| 16 | 8 | 0 | 0 | 0 | 24 |
| 11 | 9 | 0 | -1 | 0 | 20 |
| 9 | 10 | -1 | 0 | 0 | 9 |
| 3 | 11 | -1 | 1 | -1 | 7 |
| 7 | 12 | -1 | 1 | 1 | 6 |
| 10 | 13 | 1 | 0 | 0 | 71 |
| 1 | 14 | -1 | -1 | -1 | 4 |
| 5 | 15 | -1 | -1 | 1 | 13 |
| 17 | 16 | 0 | 0 | 0 | 24 |
| 15 | 17 | 0 | 0 | 0 | 36 |
| 6 | 18 | 1 | -1 | 1 | 68 |
| 14 | 19 | 0 | 0 | 1 | 33 |
| 12 | 20 | 0 | 1 | 0 | 33 |

Table 2. The design matrix

### 3.3 Conducting the experiments as per the design matrix

A total 20 number of experiments were conducted by taking the combination of input parameters as per the design matrix in table 2. All the readings were taken in single replications and the feed rate were recorded in Table 2.

### 3.4 Developing the mathematical model

The input parameters and the feed rate can be related as per the following

Feed rate= f(A,B,C),

where A,B,C are the frequency, part size and part population respectively.

The general regression equation relating the input and the output parameters is given as

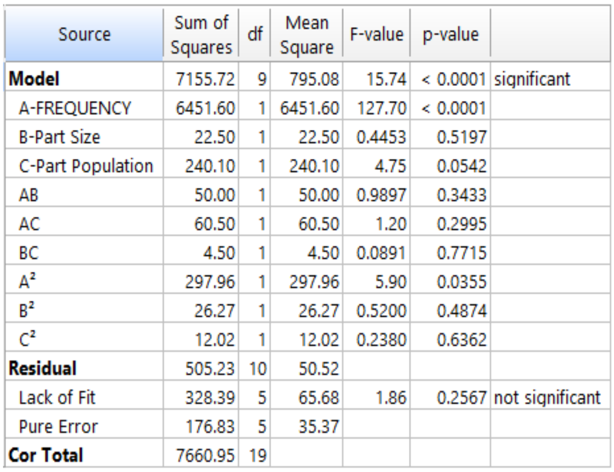
Feed Rate=$\beta_0 + \beta_1 A + \beta_2 B + \beta_3 C + \beta_{12} AB + \beta_{13} AC + \beta_{23} BC + \beta_{11} A^2 + \beta_{22} B^2 + \beta_{33} C^2$

where $\beta_0$ is model coefficient, $\beta_1$ ,$\beta_2$ ,$\beta_3$ are regression coefficient of linear term , $\beta_{12}$ , $\beta_{13}$,, $\beta_{23}$ are regression coefficient for the interaction terms $\beta_{11}$, $\beta_{22}$, $\beta_{33}$ are regression coefficient for square terms. The actual mathematical equation develop by the software is given below in the coded form.

Feed Rate= $26.94 + 25.40{*}A - 1.50{*}B + 4.90{*}C - 2.50{*}AB + 2.75{*}AC - 0.75{*}BC + 10.41{*}A^2 - 3.09{*}B^2 - 2.09{*}C^2$

### 3.5 Checking the significance of the developed model

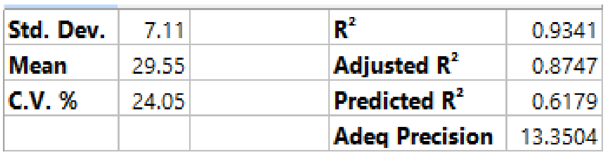
The significance of developed model is checked by ANOVA analysis whose results are given in Table 3.



| Source | Sum of Squares | df | Mean Square | F-value | p-value | |
|---|---|---|---|---|---|---|
| **Model** | 7155.72 | 9 | 795.08 | 15.74 | < 0.0001 | significant |
| A-FREQUENCY | 6451.60 | 1 | 6451.60 | 127.70 | < 0.0001 | |
| B-Part Size | 22.50 | 1 | 22.50 | 0.4453 | 0.5197 | |
| C-Part Population | 240.10 | 1 | 240.10 | 4.75 | 0.0542 | |
| AB | 50.00 | 1 | 50.00 | 0.9897 | 0.3433 | |
| AC | 60.50 | 1 | 60.50 | 1.20 | 0.2995 | |
| BC | 4.50 | 1 | 4.50 | 0.0891 | 0.7715 | |
| $A^2$ | 297.96 | 1 | 297.96 | 5.90 | 0.0355 | |
| $B^2$ | 26.27 | 1 | 26.27 | 0.5200 | 0.4874 | |
| $C^2$ | 12.02 | 1 | 12.02 | 0.2380 | 0.6362 | |
| **Residual** | 505.23 | 10 | 50.52 | | | |
| Lack of Fit | 328.39 | 5 | 65.68 | 1.86 | 0.2567 | not significant |
| Pure Error | 176.83 | 5 | 35.37 | | | |
| **Cor Total** | 7660.95 | 19 | | | | |

Table 3. ANOVA analysis of the model

The results of analysis clearly indicate that developed model is significant and lack of fit is insignificant. The fits statistics results given in table 4 with a very high value of $R^2$ proves trustworthiness of developed model within a percentage limit of 93.41%.



| | | | | |
|---|---|---|---|---|
| **Std. Dev.** | 7.11 | | **$R^2$** | 0.9341 |
| **Mean** | 29.55 | | **Adjusted $R^2$** | 0.8747 |
| **C.V. %** | 24.05 | | **Predicted $R^2$** | 0.6179 |
| | | | **Adeq Precision** | 13.3504 |

Table 4. Fit statistics

To further substantiate the validity of developed model, a scatter diagram was plotted between predicted and actual values as shown in the figure 2. The close proximity between these values clearly indicates the suitability of this model with confidence limits of 95%.

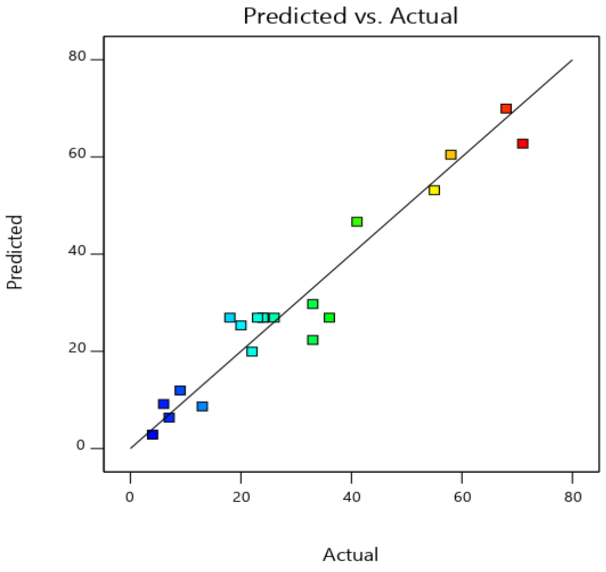


Fig 2. Scatter diagram between predicted and actual values

## 4. RESULTS AND DISCUSSIONS

The results of the investigation work carried out are depicted in graphical form shown in figures 3-8 and the explanation of direct and interaction effects of process parameters on feed rate is discussed below.

### 4.1 Direct of frequency on feed rate

The direct effect of frequency is shown in figure 3. It can be seen that feed rate increased with increase in frequency. The reason attributed can be that as frequency increased the magnitude of parts also increased resulting in faster movement of components up the spiral track.

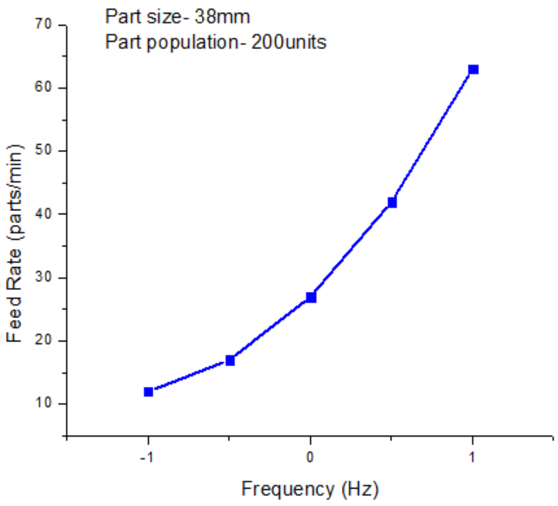


Fig. 3. Effect of frequency on feed rate

### 4.2 Direct effect of part size on feed rate

The effect of part size is shown in figure 4. It is evident that initially there is a slight increase in the feed rate with increase in the size of parts whereas, there is a decrease in the feed rate as the part size is increased further. However, the effect of part size on feed rate is almost constant.

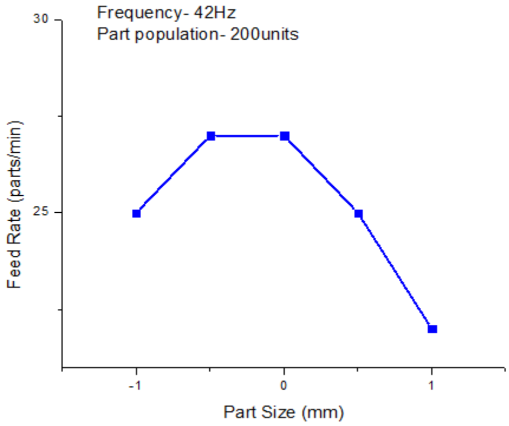


Fig. 4. Effect of part size

### 4.3 Direct effect of part population on feed rate

The effect of part population is shown in figure 5. As part population is increased it is observed that there is a marginal increase in the feed rate. The reason for this can be as the population was increased, at the entrance junction, the components were in a state of disorder and were inhibiting each other from following the main path.

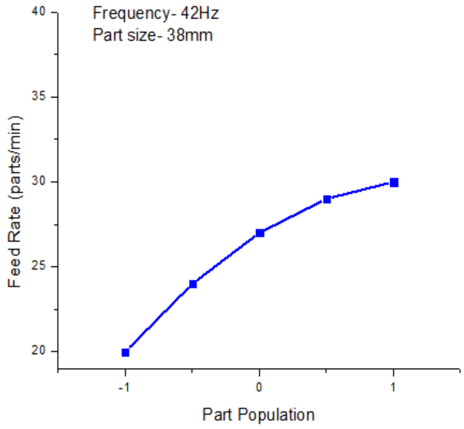


Fig. 5. Effect of part population

### 4.4 Interaction effect of frequency and part population

Interaction effect of frequency and part population on feed rate is shown in figure 6.

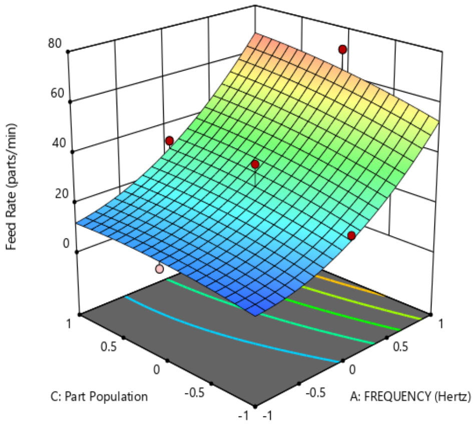


Fig. 6. Interaction effect of frequency and part population

Frequency has a positive effect on feed rate. At all part population, feed rate is increasing with the increase in frequency. However, part population has a variable effect on feed rate. For lower frequency the feed rate is almost constant, whereas there is a considerable change at higher frequency.

### 4.5 Interaction effect of part population and part size

Interaction effect of part population and part size is as shown in figure 7. When part population and part size are taken together, they have nominal effect on the feed rate. At all part population, feed rate is increasing marginally with increase in part size.

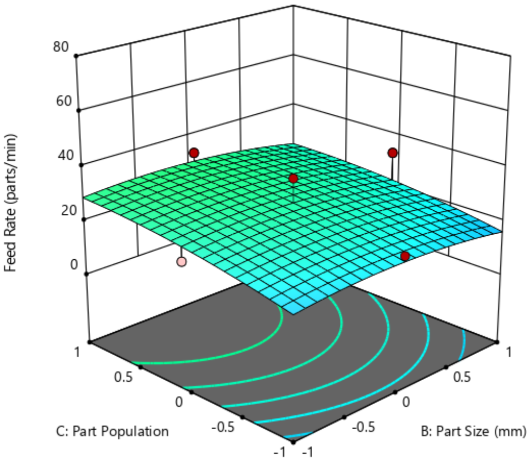


Fig. 7. Interaction effect of part population and part size

### 4.6 Interaction effect of frequency and part size

Interaction effect of frequency and part size is shown in figure 8. Frequency again has a positive effect on feed rate. At all part sizes feed rate has increased with the increase in frequency. Feed rate is almost constant at lower frequency but there is a drastic accretion at higher frequency.

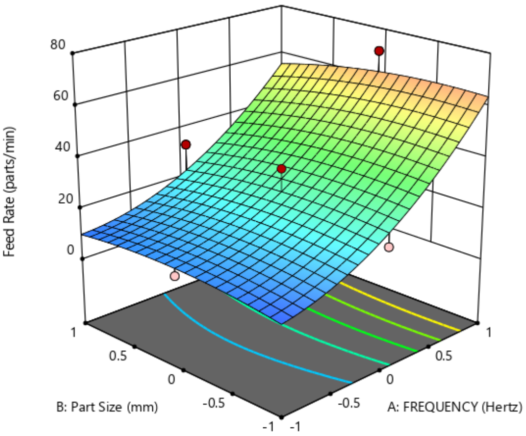


Fig. 8. Interaction effect of frequency and part size

## 5. CONCLUSIONS

1. The statistical approach of central composite face centred technique was satisfactory for the present investigation work and was able to predict the direct and interaction effect with good accuracy.

2. The direct effects indicate frequency and part size has positive effect on feed rate whereas, part population had a negative effect.

3. The interaction curve of frequency and part size shows that the minimum feed rate of 8 is attained at minimum frequency and minimum part size. Whereas, maximum feed rate of 64 is attained at maximum frequency and minimum part size.

4. The interaction curve of frequency and part population shows that the minimum feed rate of 8 is attained at minimum frequency and minimum part population. Whereas, maximum feed rate of 68 is attained at maximum frequency and maximum part population.

5. The interaction curve of part size and part population shows that the minimum feed rate of 16 is attained at maximum part size and minimum part population. Whereas, maximum feed rate of 30 is attained at 31.5mm part size and maximum part population.

## 6. REFERENCES

[1] Boothroyd, G., Dewhurst, P., Knight, W., "Product Design for Manufacture and Assembly ". CRC Press, 3rd Edition, December 2010.

[2] Jindal, U., Jain S., Khanna, P., "Graphical Analysis of a Vibratory Bowl Feeder for Clip Shaped Components". International Journal of Innovative Science, Engineering & Technology, Vol. 4, No. 2, pp. 279-283, February 2017.

[3] Pandey, T., Garg, V., Bhagat, S., Khanna, P., "Mathematical Analysis of Vibratory Bowl Feeder". International Journal of Latest Trends in Engineering and Technology, Vol. 4, No.1,pp. 315-324, May 2014.

[4] Bhagat, S., Pandey, T., Garg, V., Khanna, P., "Design, Fabrication and Analysis of Vibratory Feeder". International Journal of Research in Mechanical Engineering and Technology, Vol. 4, No.1. pp. 73-75, November 2013- April 2014.

[5] Maher, M., "Orientation Tools for Vibratory Bowl Feeders". MSc Thesis submitted to Waterford Institute of Technology, June 2010.

**Authors: Ishika Tiwari, Student, Laksha, Student, Pradeep Khanna, Associate professor,** Division of MPA Engineering, Netaji Subhas Institute of Technology, New Delhi-110078, India.
E-mail: ishi.tafsIT@gmail.com
go3laksha@gmail.com
4.khanna@gmail.com