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THE CONSTITUTIVE MODELS IN NUMERICAL SIMULATION OF STEADY-STATE METAL FORMING PROCESSES

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ABSTRACT

The theory of plasticity is in poor condition relative to linear elasticity. All existing formulations are approximate at best, and few have any connection to the fundamental (micromechanical) material mechanism responsible. For this reason, all plasticity theories must be considered empirical. Even worse, detailed testing of a plasticity theory requires extensive, careful experimentation using special machinery and precise interpretation. Accuracy of this data has a big influence on numerical calculation of metal forming processes. The very important problem in numerical simulations is defining the rigid and plastic zones in deformed material. The methods for determining these zones and the basic constitutive models used in numerical simulation of steady-state metal forming processes are analysed in this paper.

Key words: *Constitutive models, Rigid-plastic, Viscosity, Finite Volume Method*

1. INTRODUCTION

Analysis of metal forming processes has been a topic of interest to researchers in the areas of material science and mechanics for over half a century and has gained a strong impetus in the past two decades, with the advent of powerful digital computers together with sophisticated numerical tools. Such analysis is crucial, not only in providing valuable information for proper design of the manufacturing tools but also in improving the existing processes and introducing new ones. Two different approaches, namely the "flow approach" (flow formulation) and "solid approach" have emerged for the simulation of metal forming processes using different numerical methods, mainly the finite element method (FEM), [1,6,7,12] and recently the finite volume method (FVM), [2,3,8].

In the flow approach, the material is assumed to behave like a non-Newtonian viscous or visco-plastic fluid and the numerical solution is obtained by using an Eulerian reference frame (stationary mesh). This formulation uses the velocity as dependent variables. In this paper some isotropic and isothermal constitutive models used in the flow formulation which give a relation between the Cauchy stress σ_{ij} and the rate of deformation $\dot{\varepsilon}_{ij}$ are described.

2. CONSTITUTIVE MODELS

2.1. Stress-strain relations

The constitutive equation defines the relation between the stresses and strains. It is generally based on experimental observations [10]. The type of constitutive model employed depends on the material under investigation and on the applied loads. The stress-strain relation obtained from a tensile test is illustrated in Fig. 1. The material behaves in a linear elastic way up to the initial yield stress σ_{y0} with a slope E - Young's modulus. When the material is unloaded the elastic deformation ε^e is totally recovered.

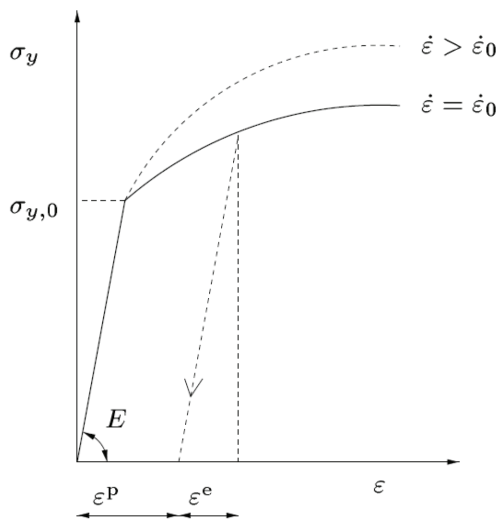


Fig.1 - Example of stress-strain curves for the one-dimensional tensile test.

Above the initial yield stress σ_{y0} the material is plastically deformed. The total deformation can be split up into an elastic and a plastic part:

$$\varepsilon = \varepsilon^e + \varepsilon^p \tag{1}$$

The yield stress σ_y increases when the material is plastically deformed and this phenomenon is called hardening. When the material is unloaded, the stress decreases again linearly according to E. The plastic deformation ε^p is not recovered, Fig. 1. Upon further plastic deformation the load has to come above the increased yield stress σ_y.

2.2. Viscoplastic behaviour

In this section the viscoplastic constitutive model is considered. This is a rate dependent material model, where the elastic deformation is neglected. In the case where residual stresses or springback phenomena are negligibly small, the elastic deformation can be omitted. In the viscoplastic model the deformation is considered to be completely plastic and the stress is rate dependent. The Cauchy stress tensor σ_{ij} is split up into the deviatoric stress tensor σ'_{ij} and the hydrostatic pressure *p*. Here the deviatoric stress tensor is described with the isothermal and isotropic Norton-Hoff model,

$$\sigma'_{ij} = 2K\left(\sqrt{3}\dot{\bar{\varepsilon}}\right)^{m-1}\dot{\varepsilon}_{ij} \tag{2}$$

In equation (2) *m* is the rate sensitivity index, $\dot{\bar{\varepsilon}}$ is the equivalent strain rate which is defined as:

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}\left(\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)} \tag{3}$$

where $\dot{\varepsilon}_{ij}$ is the strain rate tensor with components:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \tag{4}$$

where v_i (i = 1, 2, 3) is the velocity vector. *K* is the material consistency parameter and is written as,

$$K = K_0\left(\bar{\varepsilon}_0 + \bar{\varepsilon}\right)^n \tag{5}$$

where *n* is the index for the amount of hardening. The equivalent strain $\bar{\varepsilon}$ is defined by the integration of the equivalent strain rate $\dot{\bar{\varepsilon}}$:

$$\dot{\bar{\varepsilon}} = \frac{d\bar{\varepsilon}}{dt} \tag{6}$$

The material is considered to behave incompressibly, so $\dot{\varepsilon}_{ii} = 0$.
In order to be able to use the results of one-dimensional tests, the effective (equivalent) stress $\bar{\sigma}$ is needed:

$$\bar{\sigma} = \sqrt{\frac{3}{2}\left(\sigma'_{ij}\sigma'_{ij}\right)} = K\sqrt{3}\left(\sqrt{3}\dot{\bar{\varepsilon}}\right)^m \tag{7}$$

Several types of model can be derived from the Norton-Hoff model. Two types of model are considered here: the generalized Newton model and the rigid-plastic model.

Generalized Newtonian model. Generalized Newtonian behaviour is obtained when the Norton-Hoff model (2) is written in the following form,

$$\sigma_{ij}^{'} = \eta\left(\dot{\varepsilon}\right)\dot{\varepsilon}_{ij}\,, \tag{8}$$

and the shear coefficient $\eta\left(\dot{\varepsilon}\right)$ does not take into account the hardening of the material (n = 0). When the power law is applied, one can obtain:

$$\eta\left(\dot{\varepsilon}\right) = \eta_0 \bar{\dot{\varepsilon}}^{m-1} \tag{9}$$

And for m = 1 equation (9) becomes a relation for Newtonian fluid:

$$\sigma_{ij}^{'} = \eta_0 \dot{\varepsilon}_{ij} \tag{10}$$

where η_0 is a constant coefficient.
The constitutive model is independent of the total deformation $\bar{\varepsilon}$. Hence, it is not required to integrate of equation (6) to obtain the constitutive model. As a result the stress can be determined from the instantaneous velocity field. The model is said to be path independent.

Rigid-plastic behaviour. The rigid-plastic model is a special case of the Norton-Hoff model, i.e. when m = 0. In that case the equivalent stress (7) becomes:

$$\bar{\sigma} = K\sqrt{3} \tag{11}$$

From the tensile test the stress-strain relation $\bar{\sigma}\left(\bar{\varepsilon}^{p}\right)$ is obtained. Here, this result in an isotropic manner is used, that is to say the hardening is the same in all directions. Equations (11), together with the Norton-Hoff model (2), result in the Levy-Mises model:

$$\sigma_{ij}^{'} = 2\frac{\bar{\sigma}_y\left(\bar{\varepsilon}^{p}\right)}{3\bar{\dot{\varepsilon}}}\dot{\varepsilon}_{ij} \tag{12}$$

In the case of hardening effects the rigid-plastic model is path dependent, because $\bar{\varepsilon}^{p}$ is needed to describe the physical state. This means that $\bar{\dot{\varepsilon}}$ has to be integrated in time in order to be able to describe the material behaviour. The stress cannot be determined directly from the instantaneous velocity field.

3. THE FLOW FORMULATION

Steady-state processes of metal forming can be described by flow formulation which uses the velocity as dependent variables and governed by the following momentum and mass balance equations:

$$\int_S \rho v_i v_j n_j dS - \int_S \sigma_{ij} n_j dS = 0 \,, \tag{13}$$

$$\int_S \rho v_i n_j dS = 0 \,, \tag{14}$$

where ρ represents the density.

In the simulation of steady-state metal forming processes, it is common to use a rigid-(visco)-plastic constitutive model to describe the material behavior and thus neglect the elastic properties of material. The reason for this is that the elastic deformations are small compared to the very large plastic deformations that occur during the process. So, for the relation between the stress tensor and the strain-rate tensor, the Levy-Mises model (12) is used:

$$\sigma_{ij}^{'} = 2\mu\dot{\varepsilon}_{ij} \,, \tag{15}$$

where μ is the viscosity which is, according to the equation (12), defined as:

$$\mu = \frac{1}{3}\frac{\bar{\sigma}_y}{\dot{\bar{\varepsilon}}} \,, \tag{16}$$

All numerical methods consist of transforming the governing equations into a sistem of (non-linear) algebraic equation, with subsets of these approximating each conservation equation. In order to achieve this the space and the equations have to be discretised. The most popular method used for simulation of the bulk metal forming processes is finite element method (FEM). In this paper, finite volume method (FVM) based on the Eulerian mesh is used to calculate relevant parameters of steady-state metal forming processes.

4. RIGID-PLASTIC ZONES AND NUMERICAL SOLUTIONS

Equations (13) and (14) are discretised by employing a finite volume discretisation in Cartesian coordinate system, as described in [4,5].The finite volume method has dominated computational fluid dynamics for many years and has recently developed for stress analysis in solid structures. The spatial domain is discretised into a finite number of arbitrary control volumes (CV) of volume V bounded by cell faces S, with computational nodes placed in the centre of each CV.

In metal forming processes however, situations do arise in which rigid zones exist, and unloading occurs. The rigid zones are characterized by a very small value of effective strain-rate in comparison with that in the deforming body. If these portions are included within the control

volume, the value of some of the terms in discretised equation cannot be uniquely determined because the undefined value of the effective stress when the effective strain-rate approaches zero. During the calculation procedures it is possible to obtain too small values of effective strain rate in equation (16). This may cause the numerical instability, and the limiting value of the effective strain-rate $\dot{\bar{\varepsilon}}_0$, under which the material is considered to be rigid, must be introduced into calculation.
In the finite element method simulation, this difficulty is removed by assuming that in the stress – strain-rate relation the effective strain-rate is approximated by a lower limiting value $\dot{\bar{\varepsilon}}_0$, so, equation (16) becomes:

$$\mu = \frac{1}{3}\frac{\bar{\sigma}}{\left(\dot{\bar{\varepsilon}}_0 + \dot{\bar{\varepsilon}}\right)}, \tag{17}$$

with usually approximately prescribed value $\dot{\bar{\varepsilon}}_0 = 10^{-3}$, [6,11] or less, for example $\dot{\bar{\varepsilon}}_0 = 10^{-4}$ [8].

The limiting effective strain-rate, under which the material is considered to be rigid, has been introduced to improve the numerical behavior of the rigid-plastic formulation. In the FVM simulation, in the area of deformed material in which is $\dot{\bar{\varepsilon}} < \dot{\bar{\varepsilon}}_0$, material is assumed rigid, and these areas represents the so-called 'dead zones'. The function of viscosity-effective strain-rate is given in Fig. 2. The initial viscosity μ_{max}, which corresponds to $\dot{\bar{\varepsilon}}_0$ is the value at which the plastic flow begins and it is the upper value that viscosity in calculation can reach.

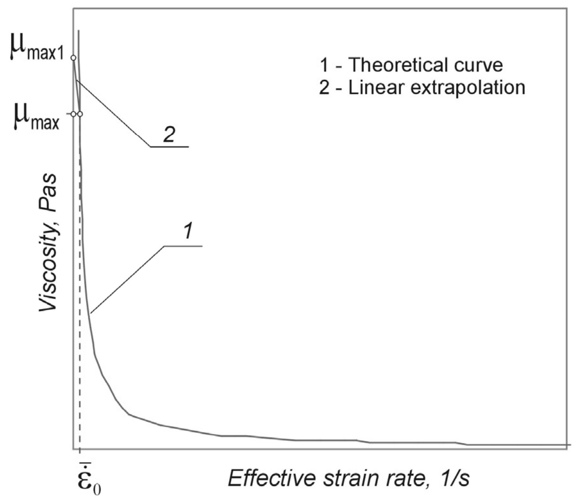


Fig. 2 - The function of viscosity.

One suggestion for definig of $\dot{\bar{\varepsilon}}_0$ in FVM calculation algorithm is given in [3] which used the initial high of workpiece H_0 and the forming velocity v_0:

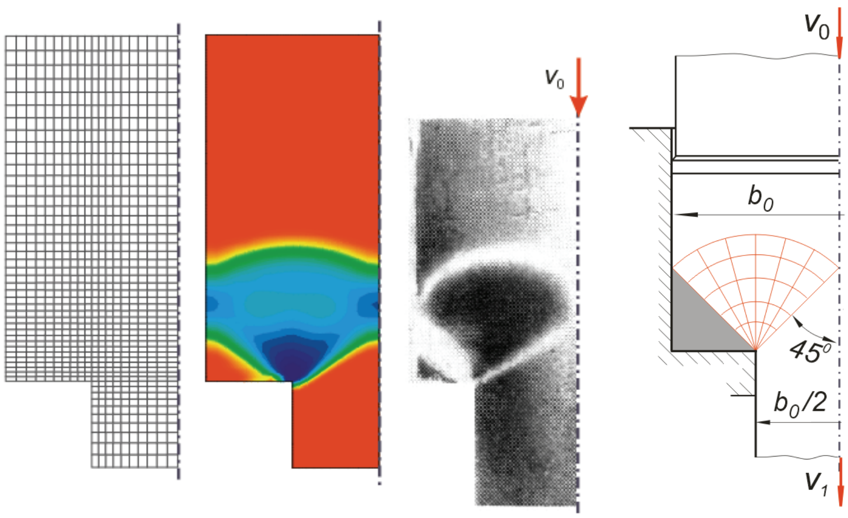
$$\bar{\dot{\varepsilon}}_0 = \frac{v_0}{H_0}, \tag{18}$$

and which gives much smaller values of $\bar{\dot{\varepsilon}}_0$ comparing with the values used in FEM simulation.

The linear extrapolation (line 2) of curve 1, given in Fig. 2, gives the better convergence of numerical procedure. This approach also requires the defining of limiting value of effective strain-rate and the additional upper value of viscosity μ_{max1}, Fig. 2. The differences between the calculated results obtained by using this extrapolation are quite small comparing with the previously described methods.

5. THE NUMERICAL EXAMPLE

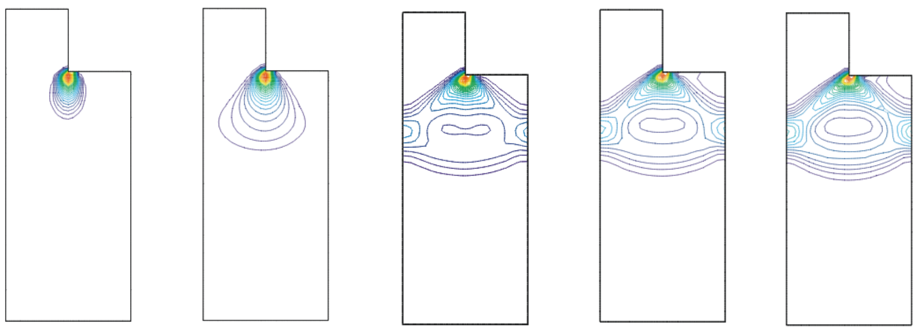
An example of plane strain forward extrusion through a flat faces die with the degree of deformation ε = 50% will be used to demonstrate the influence of limiting value of $\bar{\dot{\varepsilon}}_0$ in FVM calculation. The zero friction conditions between tool parts and workpiece is simulated. The prescribed workpiece material has yield stress σ_Y=100 MPa. The ram velocity is v_0=0,05 m/s and the initial high of workpiece is H_0=0,05 m. The discretised solution domain consists of 900 CV is given in Fig. 3a. Due to the symmetry, only the half of the solution domain is used for calculation. According to the initial high and ram velocty, the limiting value of effective strain-rate is $\bar{\dot{\varepsilon}}_0 = 1\,\mathrm{s}^{-1}$. The calculated viscosity distribution given in Fig. 3b represents at the same time the distribution of rigid and plastic areas in solution domain and this distribution is in good correlation with experimentally obtained results [9], Fig. 3c, and the solution which is obtained using slip-line field method.



(a) (b) (c) (d)

Fig. 3 - Extrusion through square die: (a) numerical mesh, (b) the viscosity distribution obtained by FVM, (c) experimentally results, [9], (d) slip-line field solution.

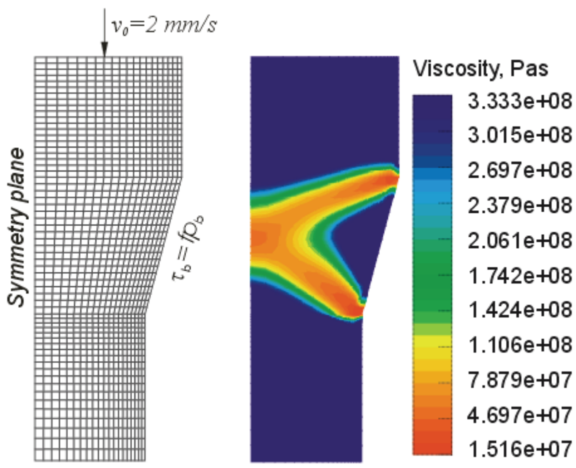
The variation of effective strain-rate fields through solution domain for different values of limiting effective strain-rate $\dot{\bar{\varepsilon}}_0$ is given in Fig. 4. It is obvious that the value $\dot{\bar{\varepsilon}}_0 = 1$ s^{-1} gives the distribution which best fit to the experimentaly results given in Fig. 3.



(a) $\dot{\bar{\varepsilon}}_0 = 5$ (b) $\dot{\bar{\varepsilon}}_0 = 2{,}5$ (c) $\dot{\bar{\varepsilon}}_0 = 1$ (d) $\dot{\bar{\varepsilon}}_0 = 0{,}5$ (e) $\dot{\bar{\varepsilon}}_0 = 0{,}1$

Fig. 4 - Distribution of effective strain rate for different values of $\dot{\bar{\varepsilon}}_0$.

The presented method of determination of borders between the rigid and plastic zones is also applied to a plane strain forward extrusion through a conical die and extrusion ratio 0,25, Fig. 5. The maximum friction is applied on the conical part of die and zero friction on other contact surfaces. The workpiece material is assumed as rigid-perfectly plastic with σ_Y =14 MPa and the prescribed ram velocity is v_0=2 mm/s. The discretised solution is presented on Fig. 5a.
The viscosity distribution given in Fig. 5b is obtained for $\dot{\bar{\varepsilon}}_0 = 0{,}9$ and this distribution is in good correlation with experimentally obtained results [9], Fig. 6.



(a) (b)

Fig. 5 - Extrusion through conical die: (a) numerical mesh, (b) viscosity distribution calculated by FVM.

The effective strain-rate given in Fig. 6 is calculated from the viscosity field given in Fig. 5b. Experimentally results in Fig. 6 are obtained by using Moire method [9].

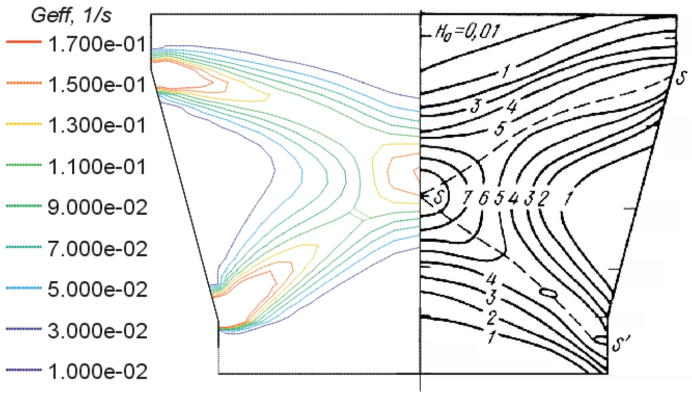


Fig. 6 - The comparison of distribution of effective strain-rate: left – FVM calculation, right – experiment [9].

6. CONCLUSION

A numerical simulation based on the FVM applied to steady state metal forming processes requires introducing a certain approximation according to the constitutive models. The definition of border between the rigid (dead) zone and the plastic zone during calculation has to be given aproximately in numerical simulations. Such approximations influence the numerical stability and the computational time. The limiting (lower) value of the effective strain rate is wide accepted criteria for determination of rigid and plastic areas of solution domains in FEM and FVM simulatons. FVM applied in flow formulaton enables obtaining the distribution of velocity components and pressure field throughout the solution domain, from which the other important variables e.g. strain-rate and stress tensor components, effective strain-rate (rigid-plastic zones) etc., can be easily calculated. The calculated results are in good correlation with theoretical considerations experiments. The values of limiting effective strain rate used in FVM simulations are smaller comparing with FEM, so the calculation has more stability, better convergence and at the same time, the calculation time is shorter.

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KONSTITUTIVNE RELACIJE KOD NUMERIČKIH SIMULACIJA KVAZISTACIONARNIH PROCESA OBLIKOVANJA METALA

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REZIME

U radu su opisani najčešće korišteni konstitutivni modeli koji se koriste pri numeričkim simulacijama procesa zapreminskog oblikovanja metala. U ovakvim situacijama opravdano je zanemarivanje elastične komponente deformacije. Stoga, za potrebe numeričkih simulacija primjenu nalaze konstitutivne relacije koje opisuju krute-(visko) plastične materijale. Polazeći od generalnog modela datog Norton-Hoffovim izrazom analizirani su specijalni slučajevi: generalni Newtonov model i Levy-Mises model. Potom je ukratko dat matematički model na kojem se temelji primjena metode konačnih volumena, a koji se sastoji od jednačine količine kretanja i jednačine kontinuiteta. Poseban problem kod numeričkih simulacija predstavlja definisanje kruto-plastičnih zona u toku deformacije. Ovaj problem se obično rješava definisanjem donje vrijednosti efektivne brzine deformacije kojoj ujedno odgovara maksimalni viskozitet deformisanog materijala. U radu je predložen metod određivanja najniže vrijednosti efektivne brzine deformacije koji daje stabilnu numeričku proceduru. Primjena metode pokazana je na dva dvodimenzionalna slučaja koji se odnose na procese istosmjernog istiskivanja koji su tretirani kao kvazistacionarni uz različite kontaktne uslove.