

## A THEORETICAL SOLUTION OF CONTACT STRESSES IN COLD HOBGING OF CONE-PUNCH

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### ABSTRACT

*Cold hobbing technology as a non-cutting process is very convenient for the production of tools in different industries, particularly in the production of die cavities. Compared with machining, sparkerosion and electrolytic treatment of tools, as well as the casting of tools, cold hobbing offers many advantages like cost saving due to a considerable reduction of process times, excellent surface finish, longer service life of the dies due to the grain flow etc.*

*Possibilities of cold hobbing technology in the manufacturing of punch-tools have been investigated in Laboratory for technology of plasticity, Faculty of Technical Sciences in Novi Sad for many years. In this paper the process of cold hobbing of cone punch has been analyzed theoretically by slab method. Using this method, identification of main process parameters has been done, and solution of axial contact stress has been obtained.*

### 1. INTRODUCTION

Cold hobbing technology is widely used in the manufacturing of tools, both for die cavities and punches. The manufacturing of die cavities by cold hobbing technology is well known process, and many solutions for different problems can be found. In this case, stress-strain state is similar to the process of backward extrusion of holes, what means that stress scheme is represented by tree compressive (negative) components. This scheme enables good flow-ability of material during the process of deformation and achievement a large amount of strain degree

On the other hand, manufacturing of punches by cold hobbing technology, is totally different process by previously mentioned, where now upper-die have interior cross-section which profile is identical to the future outer punch profile. Due to a high contact friction between upper die and billet, metal flow and process of cavities filling is very difficult, so only small dimension punches can be made in this way. In spite of great number of restriction, this method is widely used, especially in manufacturing of dies for screw industry.

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In the Laboratory of Technology of plasticity, Institute for production engineering, University of Novi Sad, process of manufacturing of punches by cold hobbing technology has been investigated theoretically and experimentally. Special attention has been paid to influence of tool and billet geometry on stress-strain state and relevant process parameters. Material formability and material flow during the process has been analyzed.

In this paper theoretical analysis of contact stress in case of cold hobbing of cone-punch, has been carried out. Contact stress was determined by using the method of solving approximate equilibrium equation (SLAB method). Due to simply geometry, this model is very convenient for general analysis of process of punch manufacturing by cold hobbing technology, where obtained solutions can be used in the cases of more complexes geometry, only with small modifications.

## 2. ANALYSIS OF CONTACT STRESS IN COLD HOBGING OF CONE-PUNCH

The scheme of cold hobbing process of cone-punch with stress components is shown on Fig.1, where three-deformation zone has been defined:

1. Inside zone - (I)
2. Outer zone - (II)
3. Inter-zone - (III)

Analysis of stress-strain state has been accomplished by using next assumptions and simplifications.

- The billet is axial-symmetric
- All plane sections parallel to the axis of cylinder remain plane during the process
- Tangential contact stress  $\tau_k$  is proportional to normal stress  $\sigma_n$  and friction coefficient  $\mu$

$$\sigma_n = \mu \cdot \tau_k$$

### I - Inside zone

In accordance with the stress scheme presented on Fig.1, the basic equilibrium equation of all components in axial direction (z-axis) is:

$$\begin{aligned} \sigma_z \cdot r^2 \cdot \pi - (\sigma_z + d\sigma_z) \cdot (r - dr)^2 \cdot \pi - \sigma_n \cdot 2\pi \cdot r \frac{dr}{\sin \alpha} \sin \alpha - \\ - \tau_k \cdot 2\pi \cdot r \frac{dr}{\sin \alpha} \cos \alpha = 0 \end{aligned} \quad (1)$$

After transformations of equation (1) by using before mentioned presumptions and simplifications, and staying to first order of small quantities, the following expression has been obtained:

$$\sigma_z + d\sigma_z \frac{r}{2dr} - \sigma_n \cdot (1 + \mu \cdot \operatorname{ctg} \alpha) = 0 \quad (2)$$

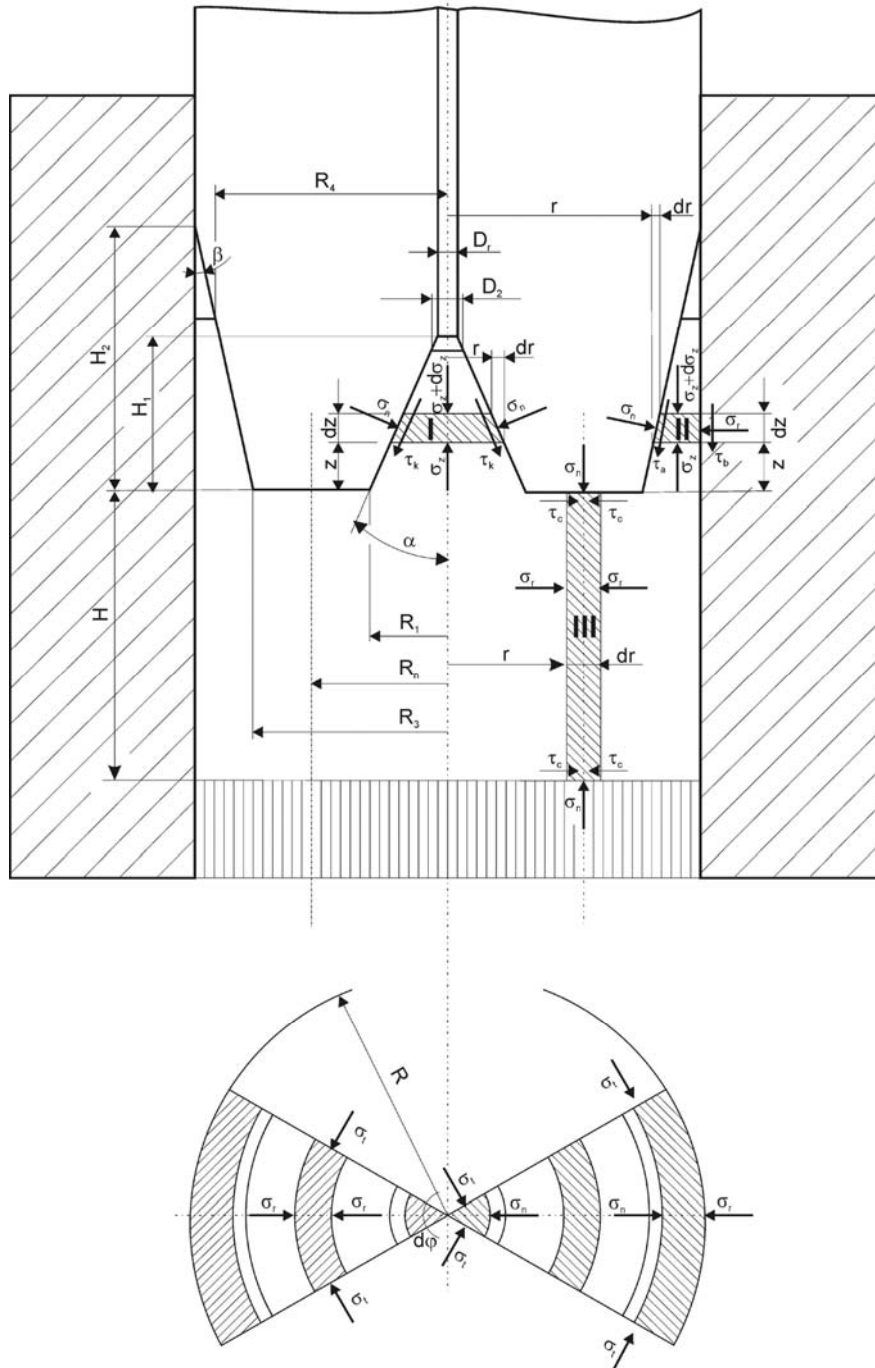


Fig.1 – Cold hobbing of cone punch - scheme of the process

By using the yield criteria elimination of normal contact stress from upper equation can be achieved. For the assumed stress scheme presented on Fig. 1, yield criteria in this zone is:

$$\sigma_n = \sigma_z + K$$

where is  $K$  – effective stress, so final differential equilibrium equation in zone I has the following form:

$$\frac{d\sigma_z}{K(I + \psi) + \sigma_z \psi} = \frac{2dr}{r} \quad (3)$$

where is:

$$\psi = \mu \cdot ctg\alpha$$

General solution of equation (3) is:

$$\sigma_z = \frac{C_1 \cdot r^{2-\psi} - K \cdot (I + \psi)}{\psi} \quad (4)$$

Constant  $C_1$  can be determined by using boundary condition, where on the free surface, component  $\sigma_z$  must be zero, i.e.: for  $r=R_2 \rightarrow \sigma_z = 0$ , hence:

$$C_1 = \frac{K \cdot (I + \psi)}{R_2^{2-\psi}} \quad (5)$$

Substituting (5) in (4), final solution for distribution of the axial stress  $\sigma_{z_I}$  in zone I is:

$$\sigma_{z_I} = \frac{K \cdot (I + \psi)}{\psi} \left[ \left( \frac{r}{R_2} \right)^{2-\psi} - I \right] \quad (6)$$

## II- Outer zone

The basic differential equilibrium equation in axial direction (z-axis) is:

$$\begin{aligned} \sigma_z (R^2 - r^2) \cdot \pi - (\sigma_z + d\sigma_z) [R^2 - (r - dr)^2] \cdot \pi - \sigma_n \cdot 2\pi \cdot r \frac{dr}{\sin \beta} \sin \beta - \\ \tau_a \cdot 2\pi \cdot r \frac{dr}{\sin \beta} \cos \beta - \tau_b \cdot 2\pi \cdot R \frac{dr}{tg \beta} = 0 \end{aligned} \quad (7)$$

As:

$\tau_a = \mu \cdot \sigma_n$  and  $\tau_b = \mu \cdot \sigma_r$ , expression (7) is reduced to:

$$2\sigma_z \cdot r dr + d\sigma_z \cdot (R^2 - r^2) - 2\sigma_n \cdot r dr \cdot (1 + \mu \cdot \text{ctg}\beta) - 2\sigma_r \cdot \mu \cdot \text{ctg}\beta \cdot R dr = 0$$

In above expression three unknown stress components:  $\sigma_z$ ,  $\sigma_r$ ,  $\sigma_n$  are present. Elimination of components  $\sigma_r$ , can be achieved by using differential equilibrium equation for radial direction (r-axis):

$$\begin{aligned} & \sigma_n \cdot r d\varphi \frac{dr}{\sin\beta} \cos\beta - \mu \cdot \sigma_n \cdot r \cdot d\varphi \frac{dr}{\sin\beta} \sin\beta + \\ & 2\sigma_t (R-r) \frac{dr}{\text{tg}\beta} \sin\left(\frac{d\varphi}{2}\right) - \sigma_r \cdot R \cdot d\varphi \frac{dr}{\text{tg}\beta} = 0 \end{aligned} \quad (8)$$

with condition that  $\sigma_r = \sigma_t$  (axial-symmetric stress state), what gives:

$$\sigma_r = \sigma_n \cdot (1 - \mu \cdot \text{tg}\beta) \quad (9)$$

Yield criteria for this zone is:

$$\sigma_n = K + \sigma_z \text{ i.e. } d\sigma_z = d\sigma_n \quad (10)$$

By substituting (9) and (10) in (8) final shape of differential equilibrium equation for zone II is obtained:

$$\frac{d\sigma_z}{dr} = -\frac{2r}{R^2 - r^2} \left\{ (\sigma_z + K) \cdot \left[ 1 + \lambda + \lambda \frac{R}{r} - \mu^2 \right] + \sigma_z \right\} \quad (11)$$

where is:  $\lambda = \mu \cdot \text{ctg}\beta$

To solve the differential equation (11) it is necessary to apply numerical integration, where the result is interpolating function, which can be used for the further calculations. Boundary condition for zone II, necessary for numerical integration procedure, is:

$$\text{for } r=R_4 \rightarrow \sigma_{z_{III}} = 0 \quad (12)$$

### III – Inter zone

From this zone (bounded by  $R_1$  and  $R_3$ , Fig.1) material flows simultaneously in two directions: to inside zone (zone I) and to outside zone (zone II). The surface where material is divided and from which it flows left and right is called neutral surface, and it is determined by neutral radius  $R_n$ . This radius is changeable during the process of deformation, and its position is mostly depended

on friction condition in both zones. As the neutral surface is the singular point, two regions with current radius  $r$ :

$$R_1 < r < R_n \quad (13)$$

and

$$R_n < r < R_3 \quad (14)$$

must be separately analyzed.

However, the function of normal stress distribution over the zone III is continual, what means that normal stress must be uniformly determined at any point of this zone, including the point of  $R_n$ . Based upon this, the position of neutral radius can be easily obtain by equalizing expressions of normal stress for both regions.

Basic differential equilibrium equation for both regions of zone III has the same shape, but difference is only in a member that comes from shear stress. According to Fig.1, it follows that:

$$\sigma_r \cdot r \cdot h \cdot d\varphi - (\sigma_r + d\sigma_r) \cdot (r + dr) \cdot h \cdot d\varphi + 2\sigma_t \cdot \sin \frac{d\varphi}{2} \cdot h \cdot dr \pm 2\tau_c \cdot r \cdot dr \cdot d\varphi = 0 \quad (15)$$

Last member with positive value are related to region where

$$R_1 < r < R_n$$

and contrary, negative value member are related to

$$R_n < r < R_3$$

To solve the expression (15) next relations between stress components can be established:

- $\sigma_r = \sigma_t$  - the condition of axial-symmetry
  - $\tau_c = \mu \cdot \sigma_n$  - the initial presumption
  - $\sigma_n - \sigma_r = K$  -yield criteria
- (16)

Substituting (16) into (15) it is reduce to:

$$\frac{d\sigma_n}{\sigma_n} = \pm \frac{2\mu}{h} \cdot r \quad (17)$$

General solution of (17) is:

$$\ln \sigma_n = \pm \frac{2\mu}{h} r + C_3 \quad (18)$$

For the determination of constant  $C_3$ , boundary condition in both regions of zone III must be considered.

**Case:**  $R_I < r < R_n$

The stresses on the boundary between zone III and I (in direction of axis z) must be equal for both zones, so boundary condition has the form:

$$\text{for } r=R_I \rightarrow \sigma_{n_{III}} = \sigma_{z_I(r=R_I)} \quad (19)$$

where  $\sigma_{z_I(r=R_I)}$  is value of  $\sigma_z$  stress in zone I, at point  $r=R_I$ , i.e.:

$$\sigma_{z_I(r=R_I)} = \frac{K \cdot (I + \psi)}{\psi} \left[ \left( \frac{R_I}{R_2} \right)^{2\psi} - I \right] \quad (20)$$

Substituting the boundary condition (19) into (20), it is obtained:

$$\sigma_{n_{III}(r < R_n)} = \sigma_{z_I(r=R_I)} \cdot e^{\frac{2\mu}{h}(r-R_I)} = \frac{K \cdot (I + \psi)}{\psi} \left[ \left( \frac{R_I}{R_2} \right)^{2\psi} - I \right] \cdot e^{\frac{2\mu}{h}(r-R_I)} \quad (21)$$

**Case:**  $R_n < r < R_3$

In this case, the boundary condition is:

$$\text{for } r=R_3 \rightarrow \sigma_{n_{III}(r < R_n)} = \sigma_{z_{II}} \quad (22)$$

where  $\sigma_{z_{II}}$  is the value of stress  $\sigma_z$  in zone II at  $r=R_3$ .

The illustrations of theoretical solution of stress distribution in axial direction (z –axes) for different angles are shown in Fig.2, Fig.3 and Fig.4.

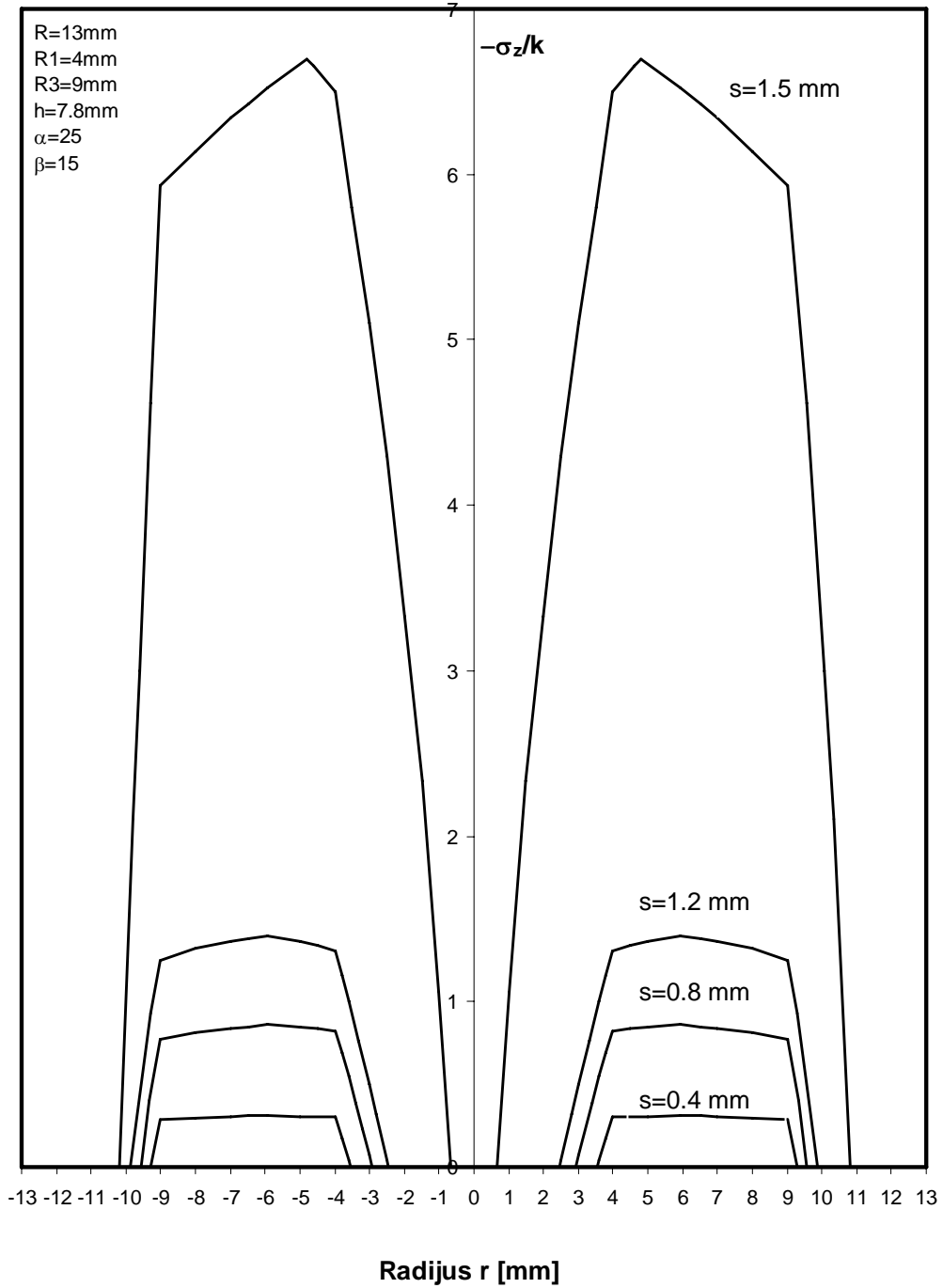


Fig.2 – Distribution of axial stress for  $\alpha=25^\circ$



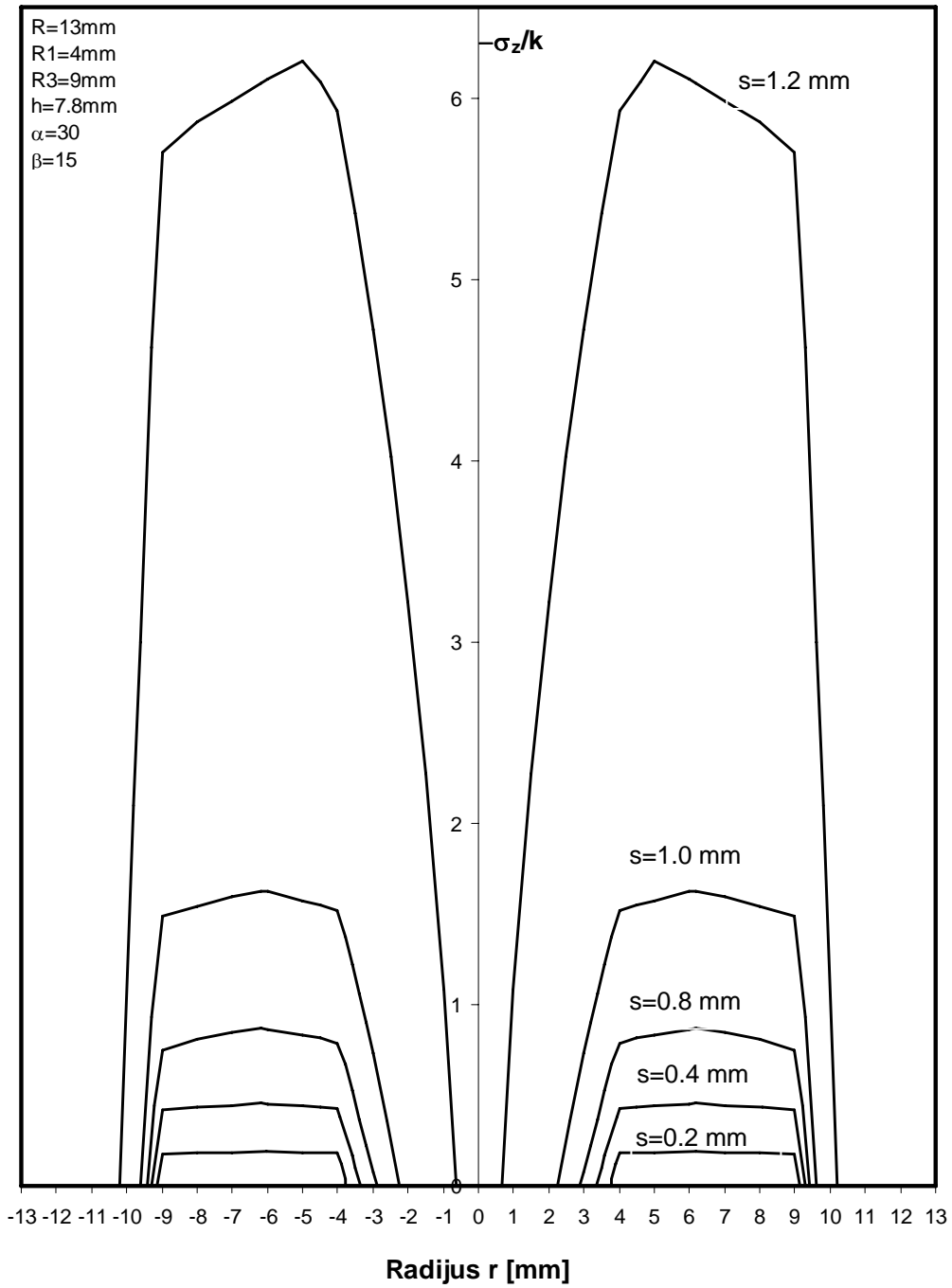


Fig.3 – Distribution of axial stress for  $\alpha=30^\circ$

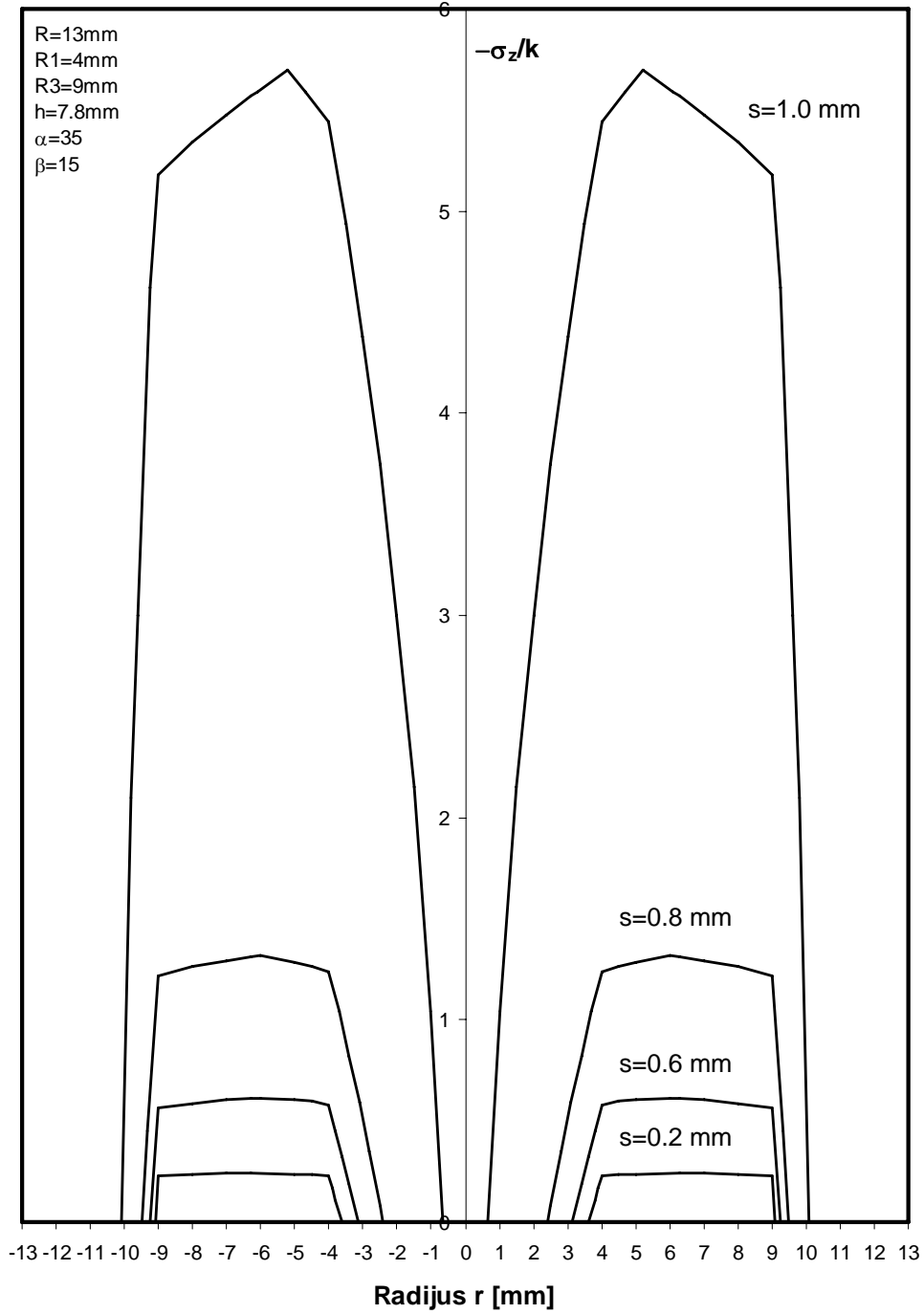


Fig.4 – Distribution of axial stress for  $\alpha=35^\circ$

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### 3. CONCLUSION

Cold hobbing technology is very convenient process for manufacturing of variety types of tools. Although this technology is used mostly for production of die cavity, punch-tools can be obtained successfully too.

The analysis conducted in this paper has shown that in the process of cold hobbing of cone punch following parameters have significant influence on stress distribution and process parameters:

- tool geometry ( $\alpha$ ,  $\beta$ ,  $R_1$ ,  $R_3$ ),
- billet geometry ( $R$ ),
- stroke (radius  $r$ )
- coefficient of friction ( $\mu$ )
- material ( $K$ )

By analyzing the way of how axial stress changes during the process of deformation it can be concluded that the form of the curve remains almost unchangeable, but the value of stress rises quickly with stroke increase. The point of maximum value is determined by means of neutral radius (Fig.2 - Fig.4). Experimental investigation has shown that at the beginning of the process neutral radius changes slowly, while by reaching the end of the process it moves rapidly to the center of the billet.

By analyzing the diagrams of axial stress (shown on Fig 2 - 4), it can be concluded that angle of cone punch ( $\alpha$ ), has dominant influence on stress value and its distribution. Increase of this angle (at the same stroke), generates the stress value increase. In the same time deformation zone becomes larger. As " $\sigma_z$ " is negative (pressure stress), the higher value of this stress enable better formability and material flow. Hereby, a problem of "corner filing" is less present, which means that more accurate dimensions of cone punch can be obtained.

The change of angle  $\beta$  has similar impact on the stress distribution as above described angle  $\alpha$ . However, its influence is less than influence of angle  $\alpha$ , and is limited by value of  $30^\circ$ . Increase of angle  $\beta$  over this value causes no significant effects on the axial stress, but it leads to the weakness of the tool, which can generate tool damage.

Contact friction significantly influences the stress state and material flow, so it is very important that value of coefficient of friction is kept as low as possible. For the calculation in this paper, it was assumed as  $\mu=0.1$ .

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## **TEORIJSKO REŠENJE ZA KONTAKTNE NAPONE U PROCESU IZRADE KONUSNOG ŽIGA HLADNIM UTISKIVANJEM**

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### **REZIME**

*Hladno utiskivanje je metoda tehnologije obrade deformisanjem koja se veoma uspešno koristi za izradu alata za potrebe različitih industrijskih grana, prvenstveno gravura. U poredjenju sa ostalim postupcima izrade alata kao što su obrada rezanjem, elektroerozija, livenje i dr., ova metoda ima čitav niz prednosti. Ovaj postupak pre svega omogućava veliku produktivnost pri izradi alata čime se znatno snižava njihova cena, dok je radni vek ovako izradjenog alata, zbog povećanja mehaničkih karakteritika polaznog materijala, nekoliko puta duži od alata dobijenog nekim drugim postupkom.*

*Pored izrade gravura u alatima ovaj postupak se može uspešno koristiti i za izradu žigova. Upravo mogućnost primene tehnologije hladnog utiskivanja pri izradi različitih žigova predmet je dugogodišnjih istraživanja koja se sprovode u Laboratoriji za obradu deformisanjem Fakulteta tehničkih nauka u Novom Sadu.*

*U ovom radu dat je prikaz teoretske analize kontaktnih napona, u slučaju izrade konusnog žiga, pomoću metode ravnih preseka. U okviru ove analize izvršena je indentifikacija glavnih parametara procesa i dobijeno je rešenje za raspored aksijalnog napona po preseku obratka. Razmatran je uticaj pojedinih faktora, pre svega geometrije alata i geometrije pripremljena, na veličinu i raspored ovog napona, kao i njihov uticaj na tečenje metala u toku procesa deformisanja. Utvrđeno je da geometrija alata ima dominantan uticaj na raspored i veličinu aksijalnog napona*