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FINITESIMAL DEFORMATION IN A TRANSVERSELY ISOTROPIC THIN ROTATING DISC WITH RIGID SHAFT

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ABSTRACT

Seth's transition theory is applied to the problems of finitesimal deformation in a transversely isotropic thin rotating disc with rigid shaft. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to transversely isotropic material and isotropic materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca's yield condition. It has been observed that rotating disc made of isotropic material required high percentage increase in angular speed to become fully plastic as compared to rotating disc made of transversely isotropic material. Radial stresses for isotropic material is maximum at the internal surface whereas for transversely isotropic material circumferential stresses is maximum at the external surface for fully-plastic state.
Keywords: *Disc, Shaft, Transversely Isotropic, Stresses, Displacement, Yielding*

1. INTRODUCTION

This paper is concerned with finitesimal deformation of rotating thin circular disk made of transversely isotropic material. There are many applications of rotating disks in science and engineering. As typical examples, we mention, steam and gas turbines, rotors, compressors, flywheels, computer disc drives and high speed gear engine etc. In the design of modern structures, increasing use is being made of materials which are transversely isotropic. The analysis of stress distribution in the circular disk rotating is important for a better understanding of the behavior and optimum design of structures. Solution for thin isotropic discs can be found in most of the standard elasticity and plasticity standard text books [1-8]. Güven [9] found the elastic - Plastic rotating disk with rigid inclusion under the assumption of Tresca's yield condition, its

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associated flow rule and linear strain hardening. To obtain the stress distribution, Guven matched the elastic-plastic stresses at the same radius $r = z$ of the disc. Perfect elasticity and ideal plasticity are two extreme properties of the material and the use of an ad-hoc rule like yield condition amounts to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another qualitatively different state, transition takes place. Since this transition is non-linear in character and difficult to investigate, workers have taken certain ad hoc assumptions like yield condition, incompressibility condition and a strain law, which may or may not valid for the problem. Sharma *etl*. [10] solved problems in elastic-plastic transition of transversely isotropic thin rotating disc by using Seth's transition theory. Seth's transition theory [11] does not require these assumptions and thus poses and solves a more general problem, from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points or turning points of the differential equations defining the deformed field and has been successfully applied to lager number of the problems [9-14]. Seth [12] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], \ (i = 1, 2, 3) \tag{1}$$

where n is the measure and $\overset{A}{e}_{ii}$ are the Almansi finite strain components. In this research paper, we investigate the problem of finitesimal deformation in a transversely isotropic thin rotating disc with rigid shaft by using Seth's transition theory. Results have been discussed numerically and depicted graphically.

2. MATHEMATICAL MODEL

Let us we consider a thin disc of constant density with central bore of radius a and external radius b. The annular disc is mounted on a shaft. The disc is rotating with angular speed ω about an axis perpendicular to its plane and passed through the center as shown in Fig. 1. The thickness of disc is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, that is, the axial stress T_{zz} is zero.

2.1 Boundary conditions

The disk considered in the present study having constant density and mounted on shaft. The inner surface of the disk is assumed to be fixed to a shaft. The outer surface of the disk is applied mechanical load. Thus, the boundary conditions of the problem are given by:

$$\begin{aligned} &\text{(i)} \quad r = a, \ u = 0 \\ &\text{(ii)} \quad r = b, \ T_{rr} = 0 \end{aligned} \tag{2}$$

where u and T_{rr} denote displacement and stress along the radial direction.

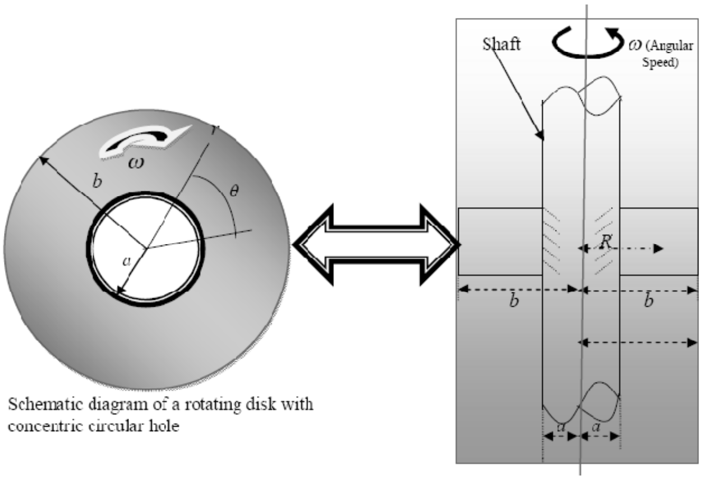

Fig.1 *– Geometry of rotating disck with shaft*

2.2 Governing equations

The components of displacement in cylindrical polar co-ordinates are given by:

$$u = r(1-\beta)\,;\ v = 0\,;\ w = dz \tag{3}$$

where β is position function, depending on $r = \sqrt{x^2 + y^2}$ only, and d is a constant.
The finite strain components are given by [12] as:

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1-\left(\beta + r\beta'\right)^2\right], \qquad \overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right],$$
$$\overset{A}{e}_{zz} = \frac{1}{2}\left[1-(1-d)^2\right], \qquad \overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0 \tag{4}$$

where $\beta' = d\beta/dr$ and meaning of superscripts "^{A}" is Almansi.
By substituting equation (4) into equation (1), the generalized components of strain are:

$$e_{rr} = \frac{1}{n}\left[1-\left(\beta + r\beta'\right)^n\right] \qquad e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right],$$
$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right] \qquad e_{r\theta} = e_{\theta z} = e_{zr} = 0 \tag{5}$$

The stress-strain relations for transversely isotropic material are given [17]:

$$T_{rr} = C_{11}e_{rr} + (C_{11} - 2C_{66})e_{\theta\theta} + C_{13}\,e_{zz}$$
$$T_{\theta\theta} = (C_{11} - 2C_{66})e_{rr} + C_{11}\,e_{\theta\theta} + C_{13}\,e_{zz}$$
$$T_{zz} = C_{13}\,e_{rr} + C_{13}e_{\theta\theta} + C_{33}\,e_{zz} = 0$$
$$T_{zr} = T_{\theta z} = T_{r\theta} = 0 \tag{6}$$

Using equation (4) in equation (6), the strain components in terms of stresses are obtained as:

$$e_{rr} \equiv \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right] = \frac{1}{E}\left[T_{rr} - \left(\frac{C_{11}C_{33}-C_{13}^2-2C_{66}C_{33}}{C_{11}C_{33}-C_{13}^2}\right)T_{\theta\theta}\right]$$
$$e_{\theta\theta} \equiv \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left[1-\beta^2\right] = \frac{1}{E}\left[T_{\theta\theta} - \left(\frac{C_{11}C_{33}-C_{13}^2-2C_{66}C_{33}}{C_{11}C_{33}-C_{13}^2}\right)T_{rr}\right]$$
$$e_{zz} \equiv \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1-\left(1-d\right)^2\right] = -\frac{1}{E}\left(\frac{C_{11}C_{33}-C_{13}^2-2C_{66}C_{33}}{C_{11}C_{33}-C_{13}^2}\right)\left[T_{rr}-T_{\theta\theta}\right]$$
$$e_{r\theta} = e_{\theta z} = e_{zr} = 0 \tag{7}$$

where $E = 4C_{66}\left(\frac{C_{11}C_{33}-C_{13}^2-C_{66}C_{33}}{C_{11}C_{33}-C_{13}^2}\right)$ is *Young's* modulus.

By substituting equations (5) into equations (6), we get:

$$T_{rr} = \frac{A}{n}\left[2-\beta^n\left\{1+\left(1+P\right)^n\right\}\right] - 2\frac{C_{66}}{n}\left[1-\beta^n\right]$$
$$T_{\theta\theta} = \frac{A}{n}\left[2-\beta^n\begin{Bmatrix}1+\\ \left(1+P\right)^n\end{Bmatrix}\right] - 2\frac{C_{66}}{n}\begin{bmatrix}1-\beta^n\\ \left(1+P\right)^n\end{bmatrix}$$
$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0 \tag{8}$$

where $A = C_{11} - \left(C_{13}^2/C_{33}\right)$.

Equations of equilibrium are all satisfied except:

$$\frac{d}{dr}\left(rT_{rr} \right) - T_{\theta\theta} + \rho\omega^2 r^2 = 0 \tag{9}$$

where ρ is the density of material.

By substituting equations (8) into equation (9), we gets a non-linear differential equation with respect to β:

$$\beta^{n+1}(1+P)^{n-1}\frac{dP}{d\beta} = \left[\frac{\rho\omega^2 r^2}{A} + \beta^n\left\{\frac{2C_{66}}{nA}\left[1+nP-(1+P)^n-P\left\{1+(1+P)^n\right\}\right]\right\}\right] \tag{10}$$

where $r\beta' = \beta P$ (P is function of β and β is function of r). The transition points of β in equation (10) are $P \to -1$ and $P \to \pm\infty$.

3. SOLUTION OF THE PROBLEM

It has been shown that the asymptotic solution through the principal stress [10, 13 - 32] leads from elastic to plastic state at the transition point $P \to \pm\infty$. If the transition function R is defined as:

$$R \equiv T_{\theta\theta} = \frac{A}{n}\left[2-\beta^n\left\{1+\left(1+P\right)^n\right\}\right]-2\frac{C_{66}}{n}\left[1-\beta^n\left(1+P\right)^n\right] \tag{11}$$

Taking the logarithmic differentiation and substitute the value of $dP/d\beta$ from equation (10) in equation (11), one gets:

$$\frac{d}{dr}(\log R) = -\frac{A}{rR}\begin{pmatrix} -\beta^n P\left\{1+\left(1+P\right)^n\right\}+\frac{2C_{66}}{nA}\beta^n-\frac{2C_{66}}{nA}\beta^n\left(1+P\right)^n+\frac{4C_{66}}{A}P\beta^n \\ +\frac{\rho\omega^2r^2}{A}-\frac{4C_{66}^2}{nA^2}\beta^n\left\{1-\left(1+P\right)^n\right\}-\frac{4C_{66}^2}{A^2}P\beta^n-\frac{2C_{66}}{A^2}\rho\omega^2r^2 \end{pmatrix} \tag{12}$$

Asymptotic value of equation (12) as $P \to \pm\infty$ and integrating, we get

$$R = K_1 r^{-C_2} \tag{13}$$

where $C_2 = 2C_{66}/A$, $A = C_{11}-\left(C_{13}^2/C_{33}\right)$ and K_1 is a constant of integration, which can be determined by the boundary condition.
Using equation (13) in equation (11), we have

$$T_{\theta\theta} = K_1 r^{-C_2} \tag{14}$$

By substituting equation (14) into equation (9), one gets:

$$T_{rr} = \frac{K_2}{r}+K_1\frac{r^{-C_2}}{1-C_2}-\frac{\rho\omega^2r^2}{3} \tag{15}$$

where K_2 is a constant of integration, which can be determined by the boundary condition.
Substituting equation (14) and (15) in second equation of (7), we get:

$$\beta = \sqrt{1-\frac{2\left(1-C_2\right)}{E}\left[\frac{\rho\omega^2r^2}{3}-\frac{K_2}{r}\right]} \tag{16}$$

Substituting equation (16) in equation (3), we get

$$u = r - r\sqrt{1-\frac{2\left(1-C_2\right)}{E}\left[\frac{\rho\omega^2r^2}{3}-\frac{K_2}{r}\right]} \tag{17}$$

where $1-C_2=\left(\frac{C_{11}C_{33}-C_{13}^2-2C_{66}C_{33}}{C_{11}C_{33}-C_{13}^2}\right)$, $E=2C_{66}\left(2-C_2\right)$ is the *Young's* modulus. By applying boundary conditions (2) in equations (15) and (17), we gets: $K_2=\rho\omega^2a^3/3$ and $K_1=\frac{\rho\omega^2\left(b^3-a^3\right)\left(1-C_2\right)}{3b.b^{-C_2}}$. Substituting values of constants K_1 and K_2 in equations (14), (15), and (17) respectively, we get the transitional stresses and displacement as:

$$T_{\theta\theta}=\frac{\rho\omega^2\left(b^3-a^3\right)\left(1-C_2\right)}{3b}\left(\frac{r}{b}\right)^{-C_2} \tag{18}$$

$$T_{rr}=\frac{\rho\omega^2}{3r}\left[\left(b^3-a^3\right)\left(\frac{r}{b}\right)^{1-C_2}+a^3-r^3\right] \tag{19}$$

$$u=r-r\sqrt{1-\frac{2\left(1-C_2\right)\rho\omega^2\left(r^3-a^3\right)}{3Er}} \tag{20}$$

From equations (18) and (19), we get

$$T_{rr}-T_{\theta\theta}=\left(\frac{\rho\omega^2}{3}\right)\left[\frac{\left(b^3-a^3\right)}{r}\left(\frac{r}{b}\right)^{1-C_2}C_2-r^2+\frac{a^3}{r}\right] \tag{21}$$

Initial Yielding: From equation (21), it is seen that $\left|T_{rr}-T_{\theta\theta}\right|$ is maximum at the internal surface (that is at $r = a$), therefore yielding will take place at the internal surface of the disc and equation (21) gives:

$$\left|T_{rr}-T_{\theta\theta}\right|_{r=a}=\left|\frac{\rho\omega^2\left(b^3-a^3\right)C_2}{3a}\left(\frac{a}{b}\right)^{1-C_2}\right|\equiv Y(say)$$

where Y is the yielding stress. Angular velocity ω_i required for initial yielding is given by:

$$\Omega_i^2=\frac{\rho\omega_i^2b^2}{Y}=\left|\frac{3ab^2}{\left(b^3-a^3\right)C_2.\left(a/b\right)^{1-C_2}}\right| \tag{22}$$

and $\omega_i=\frac{\Omega_i}{b}\sqrt{\frac{Y}{\rho}}$.

Fully-plastic state: The disc become fully plastic state $\left(C_2\to 1/2;\quad C\to 0\right)$ at the external surface and equations (21) becomes:

$$\left|T_{rr} - T_{\theta\theta}\right|_{r=b} = \left|\frac{\rho\omega^2\left(a^3 - b^3\right)}{3b}\right| \equiv Y^*$$

where Y^* is the yielding stress. The angular velocity ω_f for fully-plastic state is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \left|\frac{3b^3}{\left(a^3 - b^3\right)}\right| \tag{23}$$

where $\omega_f = \frac{\Omega_f}{b}\sqrt{\frac{Y^*}{\rho}}$. We introduce the following non-dimensional components as: $R = r/b,\ R_0 = a/b\quad \sigma_r = T_{rr}/Y,\ \sigma_\theta = T_{\theta\theta}/Y\ H^* = \mathrm{Y}^*/E\ H = Y/E$ and $U = u/b$. Elastic-plastic transitional stresses, displacement and angular speed from equations (18),(19), (20) and (22) in non-dimensional form become:

$$\sigma_\theta = \frac{\Omega_i^2\left(1 - R_0^3\right)\begin{pmatrix}1\\ -C_2\end{pmatrix}}{3}R^{-C_2} \qquad \sigma_r = \frac{\Omega_i^2}{3R}\begin{bmatrix}\left(1 - R_0^3\right)R^{1-C_2}\\ +R_0^3 - R^3\end{bmatrix}$$

$$u = r - r\sqrt{1 - \frac{2\left(1 - C_2\right)\Omega_i^2 H\begin{pmatrix}R^3\\ -R_0^3\end{pmatrix}}{3R}} \tag{24}$$

$$\Omega_i^2 = \left|\frac{3R_0}{\left(1 - R_0^3\right)C_2.R_0^{1-C_2}}\right| \tag{25}$$

Stresses, displacement and angular speed for fully- plastic state $\left(C_2 \to 0\right)$ are obtained from equations (24) and (23) become:

$$\sigma_\theta = \frac{\Omega_f^2\left(1 - R_0^3\right)}{3} \qquad \sigma_r = \frac{\Omega_f^2}{3R}\left[\left(1 - R_0^3\right)R + R_0^3 - R^3\right]$$

$$u = r - r\sqrt{1 - \frac{2\Omega_f^2 H^*\left(R^3 - R_0^3\right)}{3R}}$$

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \left|\frac{3}{\left(1 - R_0^3\right)}\right| \tag{26}$$

3.1 Isotropic case

For isotropic materials, the material constants reduce to two only, *i.e.* $C_{11} = C_{22} = C_{33}$, $C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = (C_{11} - 2C_{66})$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ .In term of constants λ and μ, these can be written as:

$$C_{12} = \lambda\,,\; C_{11} = \lambda + 2\mu \text{ and } C_{66} = \frac{1}{2}\left(C_{11} - C_{12}\right) \equiv \mu \tag{27}$$

Elastic-plastic transitional stresses are obtained by using equation (27) in equations (18) - (20), (22) as:

$$\sigma_\theta = \frac{\Omega_i^2\left(1-R_0^3\right)\left(1-C\right)}{3\left(2-C\right)} R^{-\frac{1}{2-C}} \qquad \sigma_r = \frac{\Omega_i^2}{3R}\begin{bmatrix}\left(1-R_0^3\right)R^{\frac{1-C}{2-C}} \\ +R_0^3 - R^3\end{bmatrix}$$

$$u = r - r\sqrt{1 - \frac{2(1-C)\Omega_i^2 H\left(R^3 - R_0^3\right)}{3(2-C)R}} \tag{28}$$

$$\Omega_i^2 = \left|\frac{3\left(2-C\right)R_0^{\frac{1}{2-C}}}{\left(1-R_0^3\right)}\right| \tag{29}$$

where $C = 2\mu/\left(\lambda + 2\mu\right), 1 - C_2 = \left(1 - C/2 - C\right)$.

3.2 Fully-plastic state (isotropic case)

For fully plastic state ($C \to 0$), equation (28) becomes:

$$\sigma_\theta = \frac{\Omega_f^2}{6R}\left[\left(1-R_0^3\right)R^{1/2}\right] \qquad \sigma_r = \frac{\Omega_f^2}{3R}\left[\left(1-R_0^3\right)R^{1/2} + R_0^3 - R^3\right]$$

$$U_f = R - R\sqrt{1 - H^*\left[\frac{\Omega_f^2}{3R}\begin{pmatrix}R^3 \\ -R_0^3\end{pmatrix}\right]} \tag{30}$$

The disc become fully plastic state ($C_2 \to 1/2$ or $C \to 0$) at the external surface and equations (21) becomes:

$$\left|T_{rr} - T_{\theta\theta}\right|_{r=b} = \left|\frac{\rho\omega^2\left(b^3 - a^3\right)}{6b}\right| \equiv Y^*$$

where Y^* is the yielding stress.[ ] The angular velocity ω_f for fully-plastic state is given by

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \left|\frac{6}{\left(1 - R_0^3\right)}\right| \tag{31}$$

where $R_0 = a/b$ and $\omega_f = \frac{\Omega_f}{b}\sqrt{\frac{Y^*}{\rho}}$

Equations (28) – (31) are same as given by Thakur [20].

Table 1. Elastic constants C_{ij} (in units of 10^{10} N/m^2).

Materials	C_{44}	C_{11}	C_{12}	C_{13}	C_{33}
Transversely Isotropic Material (C_2 =0.69, Beryl)	0.883	2.746	0.980	0.674	4.69
Isotropic Material ($\sigma = 0.33$ $C_2 = 0.50$, Brass)	0.999997	3.0	1.0	1.0	3.0
Transversely Isotropic Material (C_2 =0.64, Magnesium)	1.64	5.97	2.62	2.17	6.17

Table 2. Angular speed required for initial yielding and fully plastic state.

	Materials		C_2	Angular speed required for initial yielding Ω_i^2	Angular Speed required for fully-plastic state Ω_f^2	Percentage increase in angular speed $\left(\sqrt{\frac{\Omega_f^2}{\Omega_i^2}} - 1\right) \times 100$
$0.5 < R < 1$	Magnesium Beryl	Transversely Isotropic Material	0.64 0.69	3.080019 3.417748	3.428571 3.428571	0.15821 % 5.50666 %
	Brass	Isotropic Material	0.5	4.848732	6.857142	18.92072%

4. NUMERICAL ILLUSTRATION AND DISCUSSION

As a numerical example, elastic constants C_{ij} have been given in table 1 for transversely isotropic materials (Magnesium and Beryl) [19] and isotropic material [20] (Brass, $\sigma = 0.33$). It can also be seen from table 2, that rotating disc made of isotropic material (*i.e.* Brass) required high percentage increase in angular speed to become fully plastic as compared to rotating disc made of transversely isotropic materials (*i.e*. Beryal and Magnesium). Curves have been drawn in Fig.2 between angular speed Ω_i^2 required for initial yielding along the radii ratios R_0 = a/b. It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. In Fig.3 and Fig.4, curves have been drawn between stresses distribution and displacement for initial yielding and fully plastic state along the radius ratio $R = r / b$. Form fig. 3, it has been observed that isotropic material required maximum stresses and displacement as compare to transversely isotropic materials. The radial stress is maximum at the internal surface for both isotropic and transversely isotropic material. In figure 4, it is seen that radial stresses for isotropic material is maximum at the internal surface whereas for transversely isotropic material circumferential stress is maximum at the external surface. Therefore, rotating disc made of transversely isotropic material is on the safer side of the design as compared to disc made isotropic material.

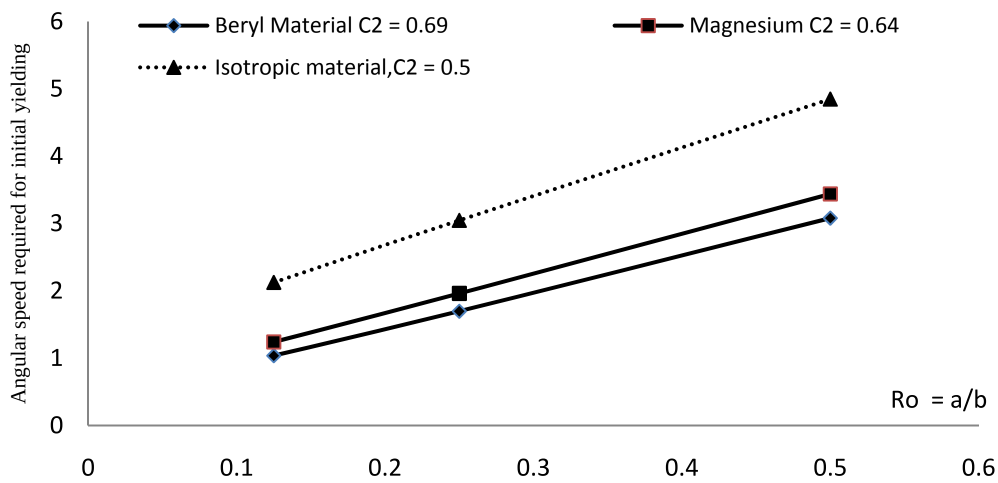

Fig. 2 - Angular speed required for initial yielding along the radii ratio R_o= a/b.

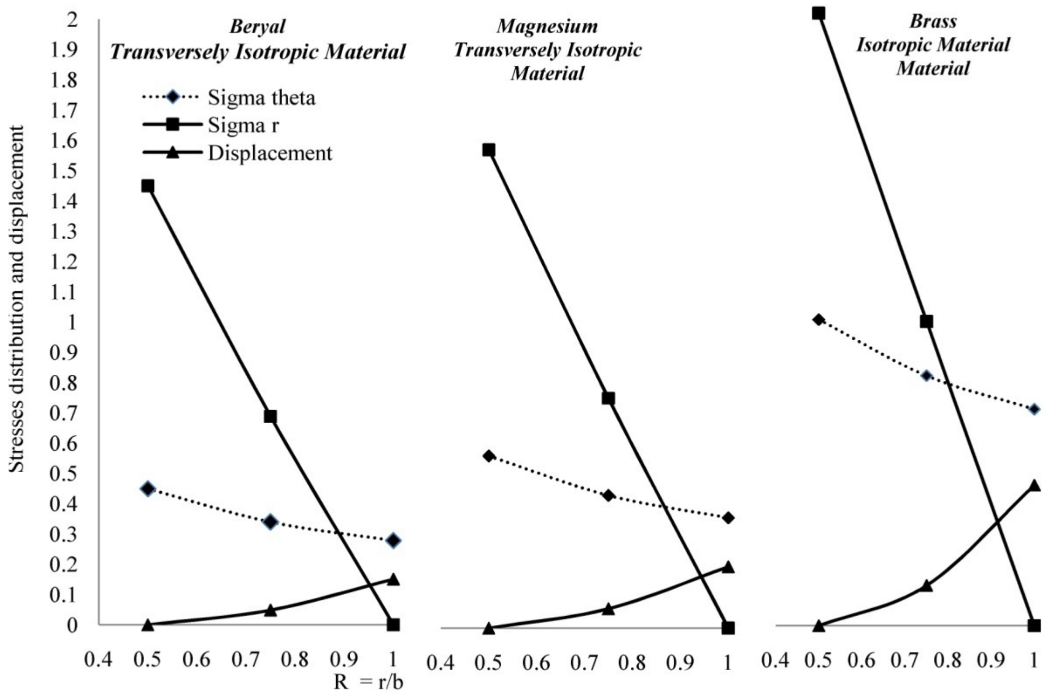

Fig-3 - *Stresses distribution and displacement for initial yielding along the radius ratio R = r/b.*

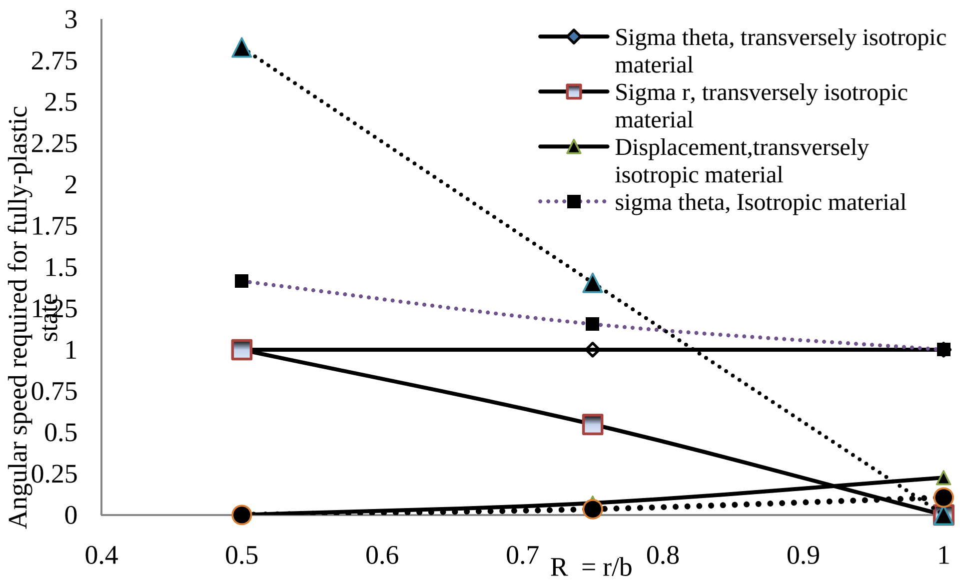

Fig.-4 - *Stresses distribution and displacement for fully-plastic state along the radius ratio R = r/b*

5. CONCLUSION

It has been observed that rotating disc made of isotropic material required high percentage increase in angular speed to become fully plastic as compared to rotating disc made of transversely isotropic material. The thin rotating disk made of transversely isotropic material yields at a higher angular speed as compared to disk made of isotropic material but isotropic disk required high percentage increase in angular speed to become fully plastic from initial yielding as compared to transversely isotropic material. Radial stresses for isotropic material is maximum at the internal surface whereas for transversely isotropic material circumferential stresses is maximum at the external surface for fully-plastic state.

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KONAČNA DEFORMACIJA U TRANSVERZALNO IZOTRPNOM TANKOM ROTIRAJUĆEM DISKU SA KRUTIM VRATILOM

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REZIME

Problem konačnih deformacija u transferzalno izotropnom tankom rotirajućem disku analiziran je pomoću Sethove tranzicione teorije. Analiza je sprovedena bez uključivanja uslova plastičnosti i zakona tečenja. Dobijeni rezultati su primenljivi kako za transverzalno izotropni materijal tako i za izotropne materijale uopšte. Ako se u analizu uvede dodatni uslov inkompresibilnosti , dobijeni obrasci za raspored napona poklapaju se sa Treskinim uslovom plastičnosti. Uočava se da rotirajući disk urađen od izotropnog materijala zahteva veći procenat povećanja ugaone brzine kako bi postao potpuno plastičan nego što je to slučaj kod diska napravljenog od transverzalno izotropnog materijala. Radijalni naponi za izotropni materijal su maksimalni na unutrašnjoj površini dok su u slučaju transverzalno izotropnih materijala cirkumferentni naponi najveći na spoljnoj površini za potpuno plastično stanje diska.
U radu je numeričkom analizom verifikovan prikazani teoretski pristup na primeru magnezijuma (transverzalno izotropan materijal) i bronze (izotropan materijal).
Ključne reči: *disk, vratilo, transferzalno izotropni materiali, naponi, pomeranja, uslov plastičnosti*