TECHNIQUE OF PARAMETRICAL AND THERMAL CALCULATION OF CALENDERS FOR PROCESSING OF PLASTIC AND RUBBER MIXES

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ABSTRACT

The technique of parametrical and thermal calculation of calenders for processing of a power-law model fluid is developed. The technique is suitable for an analysis of processing on calenders of various types with rolls with identical diameter, any friction in an inter-roll gaps, and also sequence of movement of a material on the calender. The example of calculation of the four-roll "Z" calender is elaborated.
Keywords: *power-law model fluid, calender, parameters of processing.*

1. INTRODUCTION

The further development of industry is connected with wide use of new materials, in particular thermoplastic polymeric materials. Development and manufacture of products from those materials make increased demands to the technological equipment, considering features of a design and conditions of its processing, as well as property of processed materials [1].
Different stages of calendering process of various materials (Newtonian and non-Newtonian types) are considered in detail [1],[2],[3],[4],[5], but at the same time not enough attention was given to working out of a technique of complex calculation of the calenders, allowing to analyze a various energy and power parameters of the process: temperature field of a processed material, capacity of a rolls drive, roll-separating forces, and also heat-carrier parameters in each of the rolls: a type, temperature, flow rate.
This article is focused at elaboration of a design procedure of processing of power-law materials on the calenders of different types with the rolls of identical diameter.

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2. MODELING OF CALENDERING OF POWER-LOW MODEL FLUID

Presented technique is developed on the basis of theoretical and experimental researches of flowing of the power-law model fluid in an inter-roll gap and its heat exchange on a rotating roll [1],[6] and can be used for designing a new equipment or modernization of an existing equipment. The mathematical model of flowing of the power-law model fluid in an inter-roll gap includes a differential equations of flow continuity, movement, energy, a rheological equation and an initial and boundary conditions [1],[2],[6].

During working out of mathematical model following assumptions have been introduced (Fig. 1):

- Processed material is incompressible;
- The size of the inter-roll gap on two or three order is less than diameter and length of a body of the roll;
- We neglect the material acceleration at its movement in the inter-roll gaps;
- We neglect the material weight in the inter-roll gaps;
- We neglect carrying over of heat along the bodies of rolls;
- At contact with the roll the processed material sticks to it;
- Material movement in the inter-roll gaps is plane-parallel;
- An excessive pressure in the beginning and the end of a deformation zone of the inter-roll gap equals to zero.

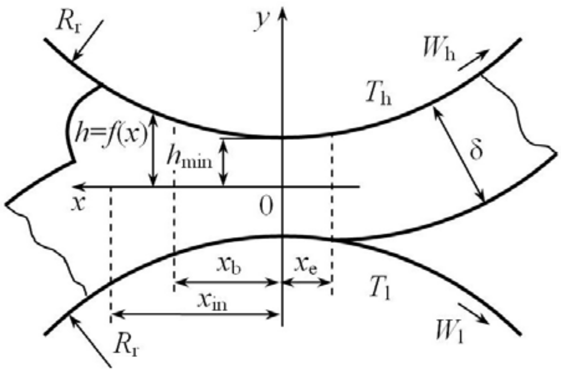

***Fig.1** – The scheme of the material flow in the inter-roll gap*: x, y *are the coordinates directed along and across of the inter-roll gap, m*; x_{in}, x_b, x_e *are coordinates of an input of the material in the inter-roll gap, the beginning and the end of the deformation zone of the inter-roll gap, m;* h, h_{min} *are half of current and half of minimum sizes of the inter-roll gap, m;* R_r *is the radius of the roll barrel, m;* δ *is a thickness of a layer of the material after its exit from the inter-roll gap, m*; W_h, W_l *are velocity of the high-velocity roll periphery and velocity of the low-velocity roll periphery, m/s;* T_h, T_l *are temperature of the high-velocity roll periphery and temperature of the low-velocity roll periphery, K*

Taking into account the accepted assumptions the differential equations describing process of material flow in the inter-roll gap, can be defined by the expressions:

$$\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}=0\ ; \tag{1}$$

$$-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xy}}{dy}=0\ ; \tag{2}$$

$$\rho c_p w_x \frac{\partial T}{\partial x}=-\frac{\partial q_y}{\partial y}+q_{\text{diss}}\ ; \tag{3}$$

$$\tau_{xy}=K\left|\frac{\partial w_x}{\partial y}\right|^n \text{sign}\left(\frac{\partial w_x}{\partial y}\right), \tag{4}$$

where w_x and w_y are components of the material velocity along axes x and y, m/s (Fig. 1 see); p is pressure in the inter-roll gap, Pa; τ_{xy} is tangential stress, Pa; T is temperature, K; ρ and c_p is density (kg/m^3) and mass thermal capacity (J/(kg·K)) of the material as a functions of temperature; q_y is a specific thermal stream in an axis direction y, W/m^2; q_{diss} is intensity of dissipation energy, W/m^3; K is consistency index, Pa·s^n; n is power low index.
A dependence of a consistence factor on temperature is defined by expression:

$$K=K_0\exp\left(-\beta\frac{T-T_0}{T_0}\right), \tag{5}$$

where K_0 is consistency index (Pa·s^n) defined at temperature T_0 (K); β is a thermal coefficient.
The initial condition on temperature is:

$$T\Big|_{x=x_{\text{in}}}=T_{\text{in}}(y). \tag{6}$$

Boundary conditions are (Fig. 1 see):
– for velocity:

$$w_x\Big|_{y=-h}=W_1=\psi W_{\text{h}}\ ; \tag{7}$$

$$w_x\Big|_{y=h}=W_{\text{h}}\ , \tag{8}$$

– for temperature:

$$T\Big|_{y=-h}=T_1\ ; \tag{9}$$

$$T\Big|_{y=h}=T_{\text{h}}\ , \tag{10}$$

where $\psi=W_1/W_{\text{h}}$ is friction factor in the inter-roll gap.
For the solution of system of the equations (1) – (5) we use dimensionless variables (coordinates) ξ and ε [1],[4],[6]:

$$\xi=\frac{x}{\sqrt{2R_{\text{r}}\,h_{\min}}}\ ;\ \varepsilon=\frac{y}{h}\text{, where }h\approx h_{\min}+\frac{x^2}{2R_{\text{r}}}=h_{\min}\left(1+\xi^2\right).$$

The solution of the system of the equations (1) – (5) at initial (6) and boundary (7) – (10) conditions allows to define:

– A temperature field of the material in any section of the inter-roll gap;
– The forces operating on each of the rolls from the material deformed in the inter-roll gap;
– The torques operating on the rolls;
– Value of dissipation energy received by the material as a result of irreversible shearing deformation in the inter-roll gap.

For definition of temperature field of the material during its stay on the roll out of the inter-roll gaps it is necessary to solve the differential equation of the non-stationary thermal conductivity in cylindrical coordinates with defined initial and boundary conditions.

For the purpose of simplification of the initial equation following assumptions are made:

– Movement of layers of the material from each other is absent;
– The sizes of the processed material out of the inter-roll gap do not change;
– The material sticks to the roll;
– Heat stream is transferred along radius of the roll by heat conductivity, thus heat stream along an axis of the roll is neglected;
– Heat stream in the material is transferred by heat conductivity according to the Fourier law

$$q_r = -\lambda \frac{\partial T}{\partial r},$$

where q_r is a specific thermal stream along an axis r coinciding with radius of the roll, W/m^2; λ is heat conductivity of the material as a function of temperature, W/(m·K).

Taking into account the accepted assumptions the equation of non-stationary heat conductivity is defined by expression

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial}{\partial r}\left(\lambda \frac{\partial T}{\partial r}\right) + \frac{\lambda}{r}\frac{\partial T}{\partial r}, \tag{11}$$

where t is time, s.

The initial condition is distribution of the material temperature to its input on the considered zone, corresponding to temperature distribution to an exit from the inter-roll gap is

$$T\big|_r = T(r). \tag{12}$$

Boundary conditions for temperature are:

$$T\big|_{r=R_\mathrm{r}} = T_{\mathrm{h(l)}}; \tag{13}$$

$$\lambda \frac{\partial T}{\partial r}\bigg|_{r=R_\mathrm{r}+\delta} = \alpha\left(T\big|_{r=R_\mathrm{r}+\delta} - T_{\mathrm{env}}\right), \tag{14}$$

where α is heat-transfer coefficient from the material surface to environment, W/(m^2·K); T_{env} is environment temperature, K.

The solution of the equation (11) under initial (12) and boundary (13) and (14) conditions allows to define a temperature field of the material during heat exchange of the material with the roll on the one surface and environment with another surface.

In further text a technique of parametrical and thermal calculation of the calenders is considered.

3. INITIAL DATA FOR CALCULATION

The initial data for calculation of the calender are:

- – arrangement scheme of the rolls;
- – scheme of movement of a formed material on the rolls;
- – sizes of the roll and its mass;
- – temperature of working surfaces of the roll (roll periphery);
- – friction factor in the inter-roll gaps;
- – thickness and width of the product removed from the calender;
- – velocity of removing product with the calender;
- – Initial temperature of the processed material at receipt in the loading inter-roll gap of the calender;
- – rheological and thermophysical properties of a processed material as temperature functions;
- – environment parameters;
- – A type of a heat-carrier in the rolls.

The technique allows:

- – define a temperature field of polymer during all time of processing on the calender;
- – calculate the forces operating on the rolls;
- – define values and directions of a torques operating from a material on the rolls and the capacities necessary for maintenance of the torques;
- – choose the heat-carrier for heating or cooling of the rolls and its parameters in the each roll (temperature and flow rate);
- – the maximum velocity of the processing material on the calender.

4. AN ALGORITHM TO CALCULATE KEY PARAMETERS OF CALENDERING

Calculation of sheet, rolled and film products on the calenders (Fig. 2) which are made by the Ukrainian industry consists in consecutive definition of following parameters:

1. The rheological and thermophysical properties of a processed material and the thermophysical properties of the rolls heat-carrier and environment as temperature functions.
2. Velocities of the rolls periphery.
3. Range of a material deformation zone in each of inter-roll gaps.
4. Sizes of the inter-roll gaps.
5. An angles of working zones of the rolls, i.e. the central angles limiting arches of a circles of the rolls which correspond to zones of deformation of the material in the inter-roll gaps, to a working surface of the each roll covered with the material, and a working surface of the each roll not covered with the material.
6. A temperature field of the material from a place of loading to a place of removing the formed product from the calender.
7. Forces operating on the rolls.
8. Torques operating from the material on the rolls and the capacities necessary for maintenance of the torques.
9. Components of power balance of every of the rolls and capacities of their heat supply systems.
10. Parameters in the each roll (temperature and flow rate).

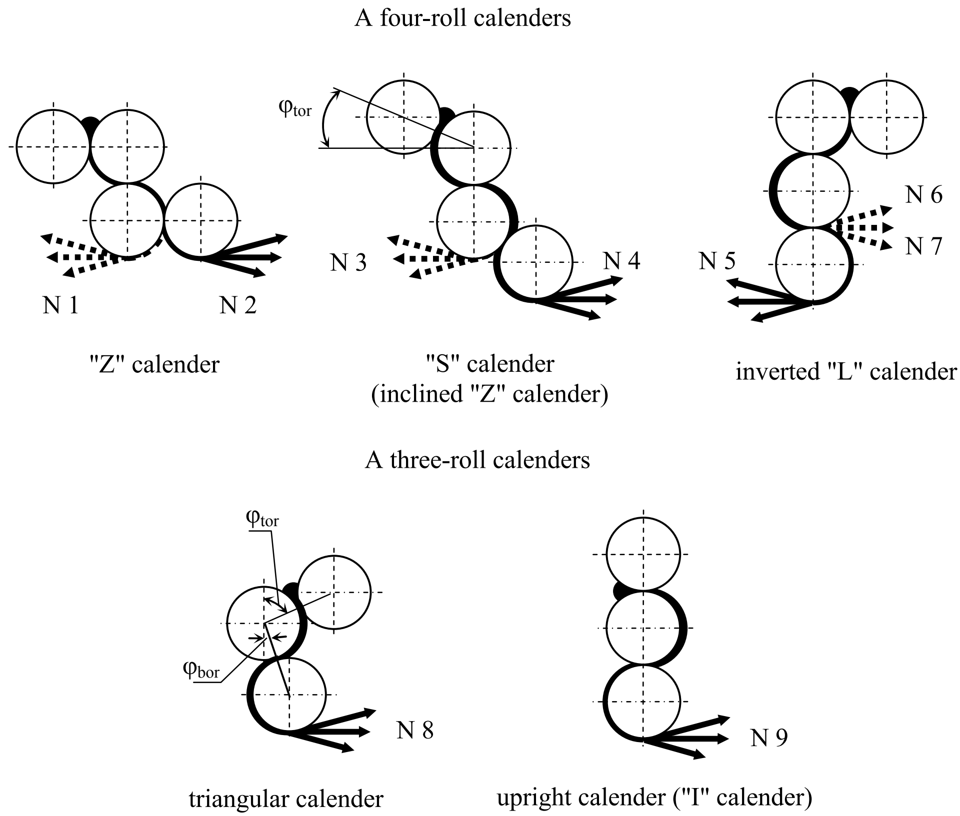

***Fig.2** – Schemes of product formation on the calenders (N 1–9 are removal schemes of a product from the calender):* φ_{tor}, φ_{bor} *are slope angles of an axis of a top overhanging roll and bottom overhanging roll*

4.1 Definition of the rheological and thermophysical properties of the processed material and the thermophysical properties of the rolls heat-carrier and environment

The rheological properties of the processed material (K_0, T_0, n and β) define the flow curves for given temperature and rate of shear at material processing.
As calendering process is not isothermal the thermophysical properties of a material from temperature it is convenient to represent dependences ($\rho(T)$, $c_p(T)$ and $\lambda(T)$ [9]) in the form of the polynomials [1].
In case of processing of a composite material for calculation of its rheological and thermophysical properties it is necessary to consider qualitative and quantitative structure of a composition [10].

4.2 Definition of rolls periphery velocities

The velocity of the roll periphery from which the formed product removes is equal:
– For configurations of the product removing N 2, 4, 5, 7–9 (Fig. 2)

$$W_m = W_{\mathrm{p}},$$

– For configurations N 1, 3 and 6

$$W_{m-1} = W_{\mathrm{p}},$$

where m is quantity of the calender rolls (rolls number from a loading inter-roll gaps); W_{p} is velocity of the product removing from the calender, m/s.
The velocity of rotation of the roll which form calibrating gap with roll from which is product removed is defined through friction factor in this gap:
– For configurations N 2, 4, 5, 7–9

$$W_{m-1} = W_m \psi_{m-1};$$

– For configurations N 1, 3 and 6

$$W_m = W_{m-1} \psi_{m-1}.$$

Definition of the roll velocities is carried out from the last roll (in the direction of moving of the formed product) to the first roll. Then, velocity of the roll number i ($i \neq m-1$ and $i \neq m$) is defined by the expression:

$$W_i = W_{i+1} \psi_i.$$

4.3 Delimitation of the deformation zone of the material in the inter-roll gaps

Borders of the deformation zone are surfaces of the rolls and also inter-roll gap sections in which excessive pressure in the processed material equals to zero. In the accepted system of dimensionless coordinates ξ and ε the surfaces of rolls correspond to coordinates $\varepsilon = \pm 1$, and sections of the beginning and the end of a zone of deformation correspond to coordinates ξ_{b} and ξ_{e}. Coordinate ξ_{e} is obtained experimentally. Its value is usually in the range between $|\xi_{\mathrm{e}}| = 0.2...0.4$ [1],[6].
The coordinate ξ_{b} corresponds to value of coordinate ξ at which equation [6] is carried out:

$$\int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{b}}} \left[\frac{|A|^n \operatorname{sign}(A) - |B|^n \operatorname{sign}(B)}{1+\xi^2} \right] d\xi = 0,$$

where $A = \left(\frac{1+2n}{n} \right) \frac{(1+\psi)(\xi^2 - \xi_{\mathrm{e}}^2)}{(1+\xi^2)^2} + \frac{1-\psi}{1+\xi^2}$, $B = -\left(\frac{1+2n}{n} \right) \frac{(1+\psi)(\xi^2 - \xi_{\mathrm{e}}^2)}{(1+\xi^2)^2} + \frac{1-\psi}{1+\xi^2}$

One of characteristics of inter-roll gap is the coordinate of an input of the material in the gap ξ_{in}, defining position of a free surface of the rotating stock which width is equal:
$2h_{\mathrm{in}\,j} = 2h_{\min j} \left(1 + \xi_{\mathrm{in}\,j}^2\right)$.
Then the coordinate ξ_{in} for a gap with number j will be defined as follows:

$$\xi_{\text{in}\,j} = \sqrt{\frac{h_{\text{in}\,j}}{h_{\min j}} - 1}$$

For the guaranteed feeding of a loading inter-roll gap of the calender with a processed material the relation $(2h_{\text{in}\,1})/(2h_{\min 1})$ is in the range from 5 to 20 that corresponds to value of coordinate ξ_{in} from 2.0 to 4.5. In the subsequent inter-roll gaps processing material is usually carried out without presence of surplus of a material on an input, i.e. under condition of $\xi_{\text{in}} = \xi_{\text{b}}$ [10].

4.4 Determination of the inter-roll gap sizes

After determination of the roll velocities it is necessary to calculate sizes of the inter-roll gaps. Mass flow rate (or mass output, kg/s) will be defined as follows:

$$G = L\rho\delta_{\text{p}}W_{\text{p}}$$

where L and δ_{p} are width and thickness of the product removed from a calender, m.
On the other hand, mass flow rate of the calender is defined by the mass flow rate of the material through the j- th inter-roll gap (j=1, 2, …, m–1) by the following expression:

$$G = L\rho\delta_{\text{p}}W_{\text{h}\,j}\left(1+\psi_j\right)h_{\min j}\left(1+\xi_{\text{e}}^2\right)$$

where $W_{\text{h}\,j}$ is velocity of the high-velocity roll from pair of rolls forming j- th inter-roll gap.
Then the size of j -th inter-roll gap can be defined as:

$$h_{\min j} = \frac{\delta_{\text{p}}W_{\text{p}}}{W_{\text{б}\,j}\left(1+\psi_j\right)\left(1+\xi_{\text{e}}^2\right)}$$

The thickness of the formed product after an exit from (j–1) - th inter-roll gap can be obtained by the expression:

$$\delta_j = h_{\min j-1}\left(1+\psi_{j-1}\right)\left(1+\xi_{\text{e}}^2\right)$$

4.5 Determination of angles of rolls working zones

For calculation of a temperature field of the material from a place of its loading to a place of its removal from calender and also for definition of heat loss from a surface of a material and a free surface of rolls it is necessary to know the angles corresponding to coordinates of an input of a material in inter-roll gap and an exit of material.
Designating an angle corresponding to coordinate of an input of the material in the j-*ŭ* inter-roll gap through $\gamma_{\xi_{\text{in}\,j}}$, and an angle corresponding to coordinate of an exit from the inter-roll gap through $\gamma_{\xi_{\text{e}\,j}}$, makes possible to write down:

$$\gamma_{\xi_{\text{in}\,j}} = \arcsin\frac{\xi_{\text{in}\,j}\sqrt{2R_{\text{r}}h_{\min j}}}{R_{\text{r}}}\,;\;\; \gamma_{\xi_{\text{e}\,j}} = \arcsin\frac{\xi_{\text{e}\,j}\sqrt{2R_{\text{r}}h_{\min j}}}{R_{\text{r}}}$$

The central angles corresponding to a free surface of the *i*- th roll $i = \left(\overline{1, m}\right)$ and a surface of the roll covered with the material ($\gamma_{fr}\ i$ and $\gamma_m\ i$ accordingly), define the dependences:
For the four-roll "Z" and "S" calenders:

$$\gamma_{fr1} = 2\pi - \gamma_{\xi_{in1}} - \gamma_{\xi_{e1}};$$

$$\gamma_{fr2} = \left(1.5\pi - \alpha_{tor}\right) - \gamma_{\xi_{in1}} - \gamma_{\xi_{e2}};$$

$$\left.\begin{aligned} \gamma_{fr3} &= \pi - \gamma_{\xi_{e2}} - \zeta \\ \gamma_{fr4} &= 2\pi - \gamma_{\xi_{in3}} - \gamma_{\xi_{e3}} \end{aligned}\right\} \text{ configurations N 1 and N 3 (Fig. 2 see)};$$

$$\left.\begin{aligned} \gamma_{fr3} &= \left(1.5\pi - \alpha_{tor}\right) - \gamma_{\xi_{in2}} - \gamma_{\xi_{e3}} \\ \gamma_{fr4} &= \left(1.5\pi - \alpha_{tor}\right) - \gamma_{\xi_{in3}} - \zeta \end{aligned}\right\} \text{ configurations N 2 and N 4};$$

$$\gamma_{m1} = 0;$$

$$\gamma_{m2} = \left(0.5\pi + \alpha_{tor}\right) - \gamma_{\xi_{e1}} - \gamma_{\xi_{in2}};$$

$$\left.\begin{aligned} \gamma_{m3} &= \pi - \gamma_{\xi_{e2}} - \gamma_{\xi_{in3}} - \gamma_{\xi_{e3}} + \zeta \\ \gamma_{m4} &= 0 \end{aligned}\right\} \text{ configurations N 1 and N 3};$$

$$\left.\begin{aligned} \gamma_{m3} &= \left(0.5\pi + \alpha_{tor}\right) - \gamma_{\xi_{e2}} - \gamma_{\xi_{in3}} \\ \gamma_{m4} &= \left(0.5\pi + \alpha_{tor}\right) - \gamma_{\xi_{e3}} + \zeta \end{aligned}\right\} \text{ configurations N 2 and N 4},$$

where α_{tor} is the angle of slope of the planes passing through axes top overhanging roll and next roll to a horizontal, rad (Fig. 2 see; for "Z" calender α_{tor}=0); ζ is angle of removing of the product from the calender concerning a horizontal, rad (upwards from a horizontal with a sign "+", downwards with a sign "–").
For the four-roll inverted "L" calender:

$$\gamma_{fr1} = 2\pi - \gamma_{\xi_{in1}} - \gamma_{\xi_{e1}};$$

$$\gamma_{fr2} = 1.5\pi - \gamma_{\xi_{in1}} - \gamma_{\xi_{e2}};$$

$$\left.\begin{aligned} \gamma_{fr3} &= \pi - \gamma_{\xi_{in2}} - \gamma_{\xi_{e3}} \\ \gamma_{fr4} &= \pi - \gamma_{\xi_{in3}} - \zeta \end{aligned}\right\} \text{ configuration N 5};$$

$$\left.\begin{aligned} \gamma_{fr3} &= \pi - \gamma_{\xi_{in2}} - \zeta \\ \gamma_{fr4} &= 2\pi - \gamma_{\xi_{in3}} - \gamma_{\xi_{e3}} \end{aligned}\right\} \text{ configuration N 6};$$

$$\left.\begin{aligned} \gamma_{fr3} &= \pi - \gamma_{\xi_{in2}} - \gamma_{\xi_{e3}} \\ \gamma_{fr4} &= 2\pi - \gamma_{\xi_{in3}} - \zeta \end{aligned}\right\} \text{ configuration N 7};$$

$$\gamma_{m1} = 0;$$

$$\gamma_{m2} = 0.5\pi - \gamma_{\xi_{e1}} - \gamma_{\xi_{in2}};$$

$$\left.\begin{aligned} \gamma_{fr3} &= \pi - \gamma_{\xi_{e2}} - \gamma_{\xi_{in3}} \\ \gamma_{m4} &= \pi - \gamma_{\xi_{e3}} + \zeta \end{aligned}\right\} \text{ configuration N 5};$$

$$\left.\begin{aligned} \gamma_{m3} &= \pi - \gamma_{\xi_{e2}} - \gamma_{\xi_{in3}} - \gamma_{\xi_{e3}} + \zeta \\ \gamma_{fr4} &= 0 \end{aligned}\right\} \text{ configuration N 6};$$

$$\left.\begin{aligned}&\gamma_{\mathrm{m3}} = \pi - \gamma_{\xi_{\mathrm{e2}}} - \gamma_{\xi_{\mathrm{in3}}}\\&\gamma_{\mathrm{fr4}} = \zeta - \gamma_{\xi_{\mathrm{e3}}} \ \text{ if } \ \zeta > \gamma_{\xi_{\mathrm{e3}}}\\&\gamma_{\mathrm{fr4}} = 0 \ \text{ if } \ \zeta \le \gamma_{\xi_{\mathrm{e3}}}\end{aligned}\right\} \quad \text{configuration N 7}.$$

For the three-roll triangular calender:

$$\gamma_{\mathrm{fr1}} = 2\pi - \gamma_{\xi_{\mathrm{in1}}} - \gamma_{\xi_{\mathrm{e1}}};$$

$$\gamma_{\mathrm{fr2}} = \pi + \alpha_{\mathrm{tor}} + \alpha_{\mathrm{bor}} - \gamma_{\xi_{\mathrm{in1}}} - \gamma_{\xi_{\mathrm{e2}}};$$

$$\gamma_{\mathrm{fr3}} = \pi + \alpha_{\mathrm{bor}} - \gamma_{\xi_{\mathrm{in2}}} - \zeta;$$

$$\gamma_{\mathrm{m1}} = 0;$$

$$\gamma_{\mathrm{m2}} = \pi - \alpha_{\mathrm{tor}} - \alpha_{\mathrm{bor}} - \gamma_{\xi_{\mathrm{e1}}} - \gamma_{\xi_{\mathrm{in2}}};$$

$$\gamma_{\mathrm{m3}} = \pi - \alpha_{\mathrm{bor}} - \gamma_{\xi_{\mathrm{e2}}} + \zeta,$$

where α_{tor} is the angle of slope of the planes passing through axes top overhanging roll and central roll to a horizontal, rad; α_{bor} is the angle of slope of the planes passing through axes bottom overhanging roll and central roll to a horizontal, rad (Fig. 2 see).

For the three-roll upright ("I") calender:

$$\gamma_{\mathrm{fr1}} = 2\pi - \gamma_{\xi_{\mathrm{in1}}} - \gamma_{\xi_{\mathrm{e1}}};$$

$$\gamma_{\mathrm{fr2}} = \pi - \gamma_{\xi_{\mathrm{in1}}} - \gamma_{\xi_{\mathrm{e2}}};$$

$$\gamma_{\mathrm{fr3}} = \pi - \gamma_{\xi_{\mathrm{in2}}} - \zeta;$$

$$\gamma_{\mathrm{m1}} = 0;$$

$$\gamma_{\mathrm{m2}} = \pi - \gamma_{\xi_{\mathrm{e1}}} - \gamma_{\xi_{\mathrm{in2}}};$$

$$\gamma_{\mathrm{m3}} = \pi - \gamma_{\xi_{\mathrm{e2}}} - \zeta.$$

The angles $\gamma_{\mathrm{m}\,i}$ are needed for calculation of a temperature field of the material and the angles $\gamma_{\mathrm{fr}\,i}$ and $\gamma_{\mathrm{m}\,i}$ are needed for thermal calculation of the calender.

4.6 Determination of material temperature

The temperature field of the material during its movement in considered inter-roll gap is computed by solving following equation [6]:

$$\left[\frac{\rho c_p W_{\mathrm{h}}}{\sqrt{R_{\mathrm{r}} h_{\min}}}\left(1 - \frac{3(1+\psi)\left(\xi^2 - \xi_{\mathrm{e}}^2\right)}{4\left(1+\xi^2\right)}\left(1-\varepsilon^2\right) - \frac{(1-\psi)}{2}(1-\varepsilon)\right)\right]\frac{\partial T}{\partial \xi} =$$

$$= \left(\frac{\lambda}{h_{\min}^2\left(1+\xi^2\right)^2}\right)\frac{\partial^2 T}{\partial \varepsilon^2} + K\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^{n+1}\left|\frac{3(1+\psi)\left(\xi - \xi_{\mathrm{e}}^2\right)}{\left(1+\xi^2\right)^2}\varepsilon + \frac{1-\psi}{1+\xi^2}\right|^{n+1}$$

taking into account initial (6) and boundary (9) and (10) conditions.
Temperature field of the material during its movement on the roll is defined by the equation (11) taking into account initial (12) and boundary (13) and (14) conditions.

The initial condition for definition of a temperature field in the loading inter-roll gap is the temperature of the material arriving on the calender. The initial condition for determination of a temperature field on each of zones of a material movement on the rolls is final distribution of temperature to the previous zone [10].

4.7 Determination of the forces operating on the rolls

The forces acting on a roll in calendering process are defined by weight of the roll and also by roll-separating forces and forces of a friction in inter-roll gaps.
The roll-separating force operating on the rolls is computed by the following expression [6]:

$$F = KLR_{\mathrm{r}}\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^{n}\int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{b}}}\int_{\xi_{\mathrm{e}}}^{\xi}\frac{|A|^{n}\operatorname{sign}(A)-|B|^{n}\operatorname{sign}(B)}{1+\xi^{2}}\,d\xi\, d\xi = 0\,,$$

where W_{h} is linear velocity of the high-velocity roll from pair of rolls which form inter-roll gap.
The vector of roll-separating force F is attached to a surface of the roll in its diametrical section in a point. Coordinate $\xi_{\mathrm{cg}\,F}$ which defines position of the area gravity centre S limited to an axis of coordinate ξ and a curve of pressure $p = f(\xi)$ in the inter-roll gap is:

$$\xi_{\mathrm{cg}\,F} = \int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{b}}}\xi\, dS \Bigg/ \int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{b}}} p\, d\xi$$

where pressure p in the inter-roll gap can be determined as [6]:

$$p = \frac{KW_{\mathrm{h}}^{n}\sqrt{2R_{\mathrm{r}}h_{\min}}}{\left(2h_{\min}\right)^{n+1}}\int_{\xi_{\mathrm{e}}}^{\xi}\frac{|A|^{n}\operatorname{sign}(A)-|B|^{n}\operatorname{sign}(B)}{1+\xi^{2}}\,d\xi$$

The angle β_F (rad) between the vector of roll-separating force F and a plane passing through axes of the rolls (i.e. an axis y) can be defined by dependence:

$$\beta_F = \arcsin\frac{\xi_{\mathrm{cg}\,F}\sqrt{2R_{\mathrm{r}}h_{\min}}}{R_{\mathrm{r}}}$$

Component F_x of the roll-separating force in a plane which passes through axes of the rolls, and also a normal component to it F_y (i.e. directed along coordinate axes x and y accordingly) is:

$$F_x = F\cos\beta_F\,;$$

$$F_y = F\sin\beta_F = F\frac{\xi_{\mathrm{cg}\,F}\sqrt{2R_{\mathrm{r}}h_{\min}}}{R_{\mathrm{r}}}$$

Thus component F_x is directed towards an exit of the material from the inter-roll gap.

As calculations show , the value of component F_y makes 99 % and more than value of the general roll-separating force (thus F_x makes to 4 % of value F), therefore for approximate calculation it is reasonable to accept that $F_y = F$.

The material deformed in the inter-roll gap leads also to occurrence of tangent stresses and accordingly the forces enclosed to working surfaces of the rolls and creating the moment of resistance to their rotation

$$P_{\mathrm{h}} = KL\sqrt{2R_{\mathrm{r}}h_{\min}}\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^n \int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{in}}} |A|^n \operatorname{sign}(A) d\xi$$

$$P_{\mathrm{l}} = -KL\sqrt{2R_{\mathrm{r}}h_{\min}}\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^n \int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{in}}} |B|^n \operatorname{sign}(B) d\xi$$

Thus, as well as roll-separating force F, it is necessary to determine forces P_{h} and P_{l} for each inter-roll gap separately.

Point of the appendix of a resultant force $P_{\mathrm{h(l)}}$ to a surface of the roll in its diametrical section defines position of the centre of gravity of the area limited to an axis of coordinate ξ and a curve of tangent stresses $\tau_{xy\mathrm{h}} = f(\xi)$ (or $\tau_{xy\mathrm{l}} = f(\xi)$) is

$$\tau_{xy\mathrm{h}} = K\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^n |A|^n \operatorname{sign}(A) \quad \text{(or } \tau_{xy\mathrm{l}} = K\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^n |B|^n \operatorname{sign}(B)\text{)}$$

The technique to determine the coordinate $\xi_{\mathrm{cg}\,P}$ (the area centre of gravity under a curve of distribution of tangent stresses for the high-velocity and low-velocity rolls) is similar to a technique of determination of coordinate $\xi_{\mathrm{cg}\,F}$. Value of an angle β_P (rad) between a vector of action of force P and an axis x (separately for the high-velocity and low-velocity rolls) is determined by the formula

$$\beta_P = \arcsin\frac{\xi_{\mathrm{cg}\,P}\sqrt{2R_{\mathrm{r}}h_{\min}}}{R_{\mathrm{r}}}$$

The value of component P_x of force P lying in a plane passing through axes of the rolls is defined by the expression:

$$P_x = P\,\xi_{\mathrm{cg}\,P}\frac{\sqrt{2R_{\mathrm{r}}h_{\min}}}{R_{\mathrm{r}}}$$

and the value of component P_y perpendicular P_x will be defined as follows

$P_y = P\cos\beta_P$.

The torques necessary for deformation of the material is:

$$M_{\mathrm{h}} = KLR_{\mathrm{r}}\sqrt{2R_{\mathrm{r}}h_{\min}}\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^{n}\int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{in}}}|A|^{n}\,\mathrm{sign}(A)d\xi\,;$$

$$M_{1} = -KLR_{\mathrm{r}}\sqrt{2R_{\mathrm{r}}h_{\min}}\left(\frac{W_{\mathrm{h}}}{2h_{\min}}\right)^{n}\int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{in}}}|B|^{n}\,\mathrm{sign}(B)d\xi$$

Vector of total effort $\boldsymbol{F}_{\mathrm{в}i\Sigma}$ acting on the i - th roll $i=\left(\overline{1,m}\right)$ is defined as the sum of vectors of gravity of the roll $\boldsymbol{G}_{\mathrm{в}\,i}$, roll-separating forces $\boldsymbol{F}_j$ and forces of friction $\boldsymbol{P}_{\mathrm{h(l)}j}$.

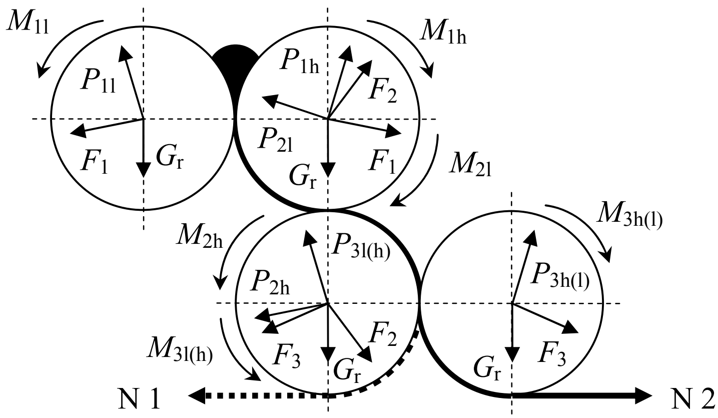

Fig.3 *- The scheme of forces acting on the four-roll "Z" calender*

Values of the forces operating on the rolls of the calenders, represented on Fig. 3, are defined by following expressions:
For four-roll "Z" and "S" calenders:

$$F_{\mathrm{r}1\Sigma} = \left[\left(F_1\sin\left(\alpha_{\mathrm{tor}}-\beta_{F_1}\right)-G_{\mathrm{r}}+P_{1\mathrm{l}}\cos\left(\alpha_{\mathrm{tor}}-\beta_{P_{1\mathrm{l}}}\right)\right)^2+\left(F_1\cos\left(\alpha_{\mathrm{tor}}-\beta_{F_1}\right)-P_{1\mathrm{l}}\sin\left(\alpha_{\mathrm{tor}}-\beta_{P_{1\mathrm{l}}}\right)\right)^2\right]^{0.5};$$

$$F_{\mathrm{r}2\Sigma} = \left[\left(F_2\cos\beta_{F_2}+P_{2\mathrm{l}}\sin\beta_{P_{2\mathrm{l}}}+P_{1\mathrm{h}}\sin\left(\alpha_{\mathrm{tor}}+\beta_{P_{1\mathrm{h}}}\right)-G_{\mathrm{r}}-F_1\sin\left(\alpha_{\mathrm{tor}}+\beta_{F_1}\right)\right)^2+\right.$$
$$\left.+\left(F_2\sin\beta_{F_2}-P_{2\mathrm{l}}\cos\beta_{P_{2\mathrm{l}}}+P_{1\mathrm{h}}\cos\left(\alpha_{\mathrm{tor}}+\beta_{P_{1\mathrm{h}}}\right)+F_1\cos\left(\alpha_{\mathrm{tor}}+\beta_{F_1}\right)\right)^2\right]^{0.5};$$

– for configurations N 1 and 3 (Fig. 2 see):

$$F_{\mathrm{r}3\Sigma} = \left[\left(F_3\sin\left(\alpha_{\mathrm{tor}}-\beta_{F_3}\right)+P_{3\mathrm{l}}\cos\left(\alpha_{\mathrm{tor}}-\beta_{P_{3\mathrm{l}}}\right)-G_{\mathrm{r}}-F_2\cos\beta_{F_2}-P_{2\mathrm{h}}\sin\beta_{P_{2\mathrm{h}}}\right)^2+\right.$$
$$\left.+\left(F_3\cos\left(\alpha_{\mathrm{tor}}-\beta_{F_3}\right)-P_{3\mathrm{l}}\sin\left(\alpha_{\mathrm{tor}}-\beta_{P_{3\mathrm{l}}}\right)+P_{2\mathrm{h}}\cos\beta_{P_{2\mathrm{h}}}-F_2\sin\beta_{F_2}\right)^2\right]^{0.5};$$

$$F_{\mathrm{в}4\Sigma} = \left[\left(P_{3\mathrm{h}}\cos\left(\alpha_{\mathrm{tor}}+\beta_{P_{3\mathrm{h}}}\right)-G_{\mathrm{r}}-F_3\sin\left(\alpha_{\mathrm{tor}}+\beta_{F_3}\right)\right)^2+\left(P_{3\mathrm{h}}\sin\left(\alpha_{\mathrm{tor}}+\beta_{P_{3\mathrm{h}}}\right)+F_3\cos\left(\alpha_{\mathrm{tor}}+\beta_{F_3}\right)\right)^2\right]^{0.5};$$

– for configurations N 2 and 4 (Fig. 2 see):

$$F_{\text{в}3\Sigma} = \Big[\Big(F_3 \sin\big(\alpha_{\text{tor}} - \beta_{F_3}\big) + P_{3\text{h}} \cos\big(\alpha_{\text{tor}} - \beta_{P_{3\text{h}}}\big) - G_{\text{r}} - F_2 \cos\beta_{F_2} - P_{2\text{h}} \sin\beta_{P_{2\text{h}}}\Big)^2 +$$
$$+ \Big(F_3 \cos\big(\alpha_{\text{tor}} - \beta_{F_3}\big) - P_{3\text{h}} \sin\big(\alpha_{\text{tor}} - \beta_{P_{3\text{h}}}\big) + P_{2\text{h}} \cos\beta_{P_{2\text{h}}} - F_2 \sin\beta_{F_2}\Big)^2\Big]^{0.5};$$

$$F_{\text{в}4\Sigma} = \Big[\Big(P_{3\text{l}} \cos\big(\alpha_{\text{вв}} + \beta_{P_{3\text{l}}}\big) - G_{\text{r}} - F_3 \sin\big(\alpha_{\text{tor}} + \beta_{F_3}\big)\Big)^2 + \Big(P_{3\text{l}} \sin\big(\alpha_{\text{tor}} + \beta_{P_{3\text{l}}}\big) + F_3 \cos\big(\alpha_{\text{tor}} + \beta_{F_3}\big)\Big)^2\Big]^{0.5}.$$

For four-roll inverted "L" calender:

$$F_{\text{r}1\Sigma} = \Big[\Big(F_1 \sin\beta_{F_1} + G_{\text{r}} - P_{1\text{l}} \cos\beta_{P_{1\text{l}}}\Big)^2 + \Big(F_1 \cos\beta_{F_1} - P_{1\text{l}} \sin\beta_{P_{1\text{l}}}\Big)^2\Big]^{0.5};$$

$$F_{\text{r}2\Sigma} = \Big[\Big(F_1 \sin\beta_{F_1} + G_{\text{r}} - F_2 \cos\beta_{F_2} - P_{1\text{h}} \cos\beta_{P_{1\text{h}}} - P_{2\text{l}} \sin\beta_{P_{2\text{l}}}\Big)^2 +$$
$$+ \Big(F_1 \cos\beta_{F_1} + F_2 \sin\beta_{F_2} + P_{1\text{h}} \sin\beta_{P_{1\text{h}}} - P_{2\text{l}} \cos\beta_{P_{2\text{l}}}\Big)^2\Big]^{0.5};$$

– for configurations N 5 and 7 (Fig. 2 see):

$$F_{\text{r}3\Sigma} = \Big[\Big(F_2 \cos\beta_{F_2} + G_{\text{r}} + P_{2\text{h}} \sin\beta_{P_{2\text{h}}} - F_3 \cos\beta_{F_3} - P_{3\text{l}} \sin\beta_{P_{3\text{l}}}\Big)^2 +$$
$$+ \Big(F_2 \sin\beta_{F_2} - P_{2\text{h}} \cos\beta_{P_{2\text{h}}} - F_3 \sin\beta_{F_3} + P_{3\text{l}} \cos\beta_{P_{3\text{l}}}\Big)^2\Big]^{0.5};$$

$$F_{\text{r}4\Sigma} = \Big[\Big[\Big(F_3 \cos\beta_{F_3} + G_{\text{r}} + P_{3\text{h}} \sin\beta_{P_{3\text{h}}}\Big)^2 + \Big(F_3 \sin\beta_{F_3} - P_{3\text{h}} \cos\beta_{P_{3\text{h}}}\Big)^2\Big]^{0.5};$$

– for configuration N 6 (Fig. 2 see):

$$F_{\text{r}3\Sigma} = \Big[\Big(F_2 \cos\beta_{F_2} + G_{\text{r}} + P_{2\text{h}} \sin\beta_{P_{2\text{h}}} - F_3 \cos\beta_{F_3} - P_{3\text{h}} \sin\beta_{P_{3\text{h}}}\Big)^2 +$$
$$+ \Big(F_2 \sin\beta_{F_2} - P_{2\text{h}} \cos\beta_{P_{2\text{h}}} - F_3 \sin\beta_{F_3} + P_{3\text{h}} \cos\beta_{P_{3\text{h}}}\Big)^2\Big]^{0.5};$$

$$F_{\text{r}4\Sigma} = \Big[\Big[\Big(F_3 \cos\beta_{F_3} + G_{\text{r}} + P_{3\text{l}} \sin\beta_{P_{3\text{l}}}\Big)^2 + \Big(F_3 \sin\beta_{F_3} - P_{3\text{l}} \cos\beta_{P_{3\text{l}}}\Big)^2\Big]^{0.5}$$

For a three-roll triangular calender:

$$F_{\text{r}1\Sigma} = \Big[\Big(F_1 \cos\big(\alpha_{\text{tor}} + \beta_{F_1}\big) - G_{\text{r}} + P_{1\text{l}} \cos\big(\alpha_{\text{tor}} + \beta_{P_{1\text{l}}}\big)\Big)^2 + \Big(F_1 \sin\big(\alpha_{\text{tor}} + \beta_{F_1}\big) - P_{1\text{l}} \sin\big(\alpha_{\text{tor}} + \beta_{P_{1\text{l}}}\big)\Big)^2\Big]^{0.5};$$

$$F_{\text{r}2\Sigma} = \Big[\Big(F_1 \cos\big(\alpha_{\text{tor}} - \beta_{F_1}\big) + G_{\text{r}} - F_2 \cos\big(\alpha_{\text{bor}} + \beta_{F_2}\big) - P_{1\text{h}} \sin\big(\alpha_{\text{tor}} - \beta_{P_{1\text{h}}}\big) - P_{2\text{l}} \cos\big(\alpha_{\text{bor}} - \beta_{P_{2\text{l}}}\big)\Big)^2 +$$
$$+ \Big(F_1 \sin\big(\alpha_{\text{tor}} - \beta_{F_1}\big) + F_2 \sin\big(\alpha_{\text{bor}} + \beta_{F_2}\big) + P_{1\text{h}} \cos\big(\alpha_{\text{tor}} - \beta_{P_{1\text{h}}}\big) - P_{2\text{l}} \sin\big(\alpha_{\text{bor}} - \beta_{P_{2\text{l}}}\big)\Big)^2\Big]^{0.5};$$

$$F_{\text{r}3\Sigma} = \Big[\Big(F_2 \cos\big(\alpha_{\text{bor}} - \beta_{F_2}\big) + G_{\text{r}} - P_{2\text{h}} \cos\big(\alpha_{\text{bor}} + \beta_{P_{2\text{h}}}\big)\Big)^2 +$$
$$+ \Big(F_2 \sin\big(\alpha_{\text{bor}} - \beta_{F_2}\big) + P_{2\text{h}} \sin\big(\alpha_{\text{bor}} + \beta_{P_{2\text{h}}}\big)\Big)^2\Big]^{0.5}.$$

For a three-roll upright ("I") calender:

$$F_{\mathrm{r1}\Sigma} = \left[\left(G_{\mathrm{r}} - F_1\cos\beta_{F_1} - P_{11}\sin\beta_{P_{11}}\right)^2 + \left(F_1\sin\beta_{F_1} - P_{11}\cos\beta_{P_{11}}\right)^2\right]^{0.5};$$

$$F_{\mathrm{r2}\Sigma} = \left[\left(F_1\cos\beta_{F_1} + G_{\mathrm{r}} + P_{\mathrm{1h}}\sin\beta_{P_{\mathrm{1h}}} - P_{21}\sin\beta_{P_{21}} - F_2\cos\beta_{F_2}\right)^2 + \right.$$
$$\left. + \left(F_1\sin\beta_{F_1} - P_{\mathrm{1h}}\cos\beta_{P_{\mathrm{1h}}} + P_{21}\cos\beta_{P_{21}} - F_2\sin\beta_{F_2}\right)^2\right]^{0.5};$$

$$F_{\mathrm{r3}\Sigma} = \left[\left(F_2\cos\beta_{F_2} + G_{\mathrm{r}} + P_{\mathrm{2h}}\cos\beta_{P_{\mathrm{2h}}}\right)^2 + \left(P_{\mathrm{2h}}\sin\beta_{P_{\mathrm{2h}}} - F_2\sin\beta_{F_2}\right)^2\right]^{0.5}.$$

The specific load operating on the i- th roll is defined by dependence:

$$q_{i\Sigma} = \frac{F_{\mathrm{r}i\Sigma}}{L}$$

The knowledge of values of the specified forces allows not only to calculate elements of a calender on durability and rigidity, but also to correctly determine the value of roll crown of calibrating inter-roll gap and value of counter bending or crossings of the roll of a calender that, finally, will provide high-quality production.

4.8 Determination of rolls drive capacity

The torques which are transferred by universal spindles of a calender drive to high-velocity and low-velocity rolls can be defined by the dependence:

$$M_{\mathrm{h(l)}\Sigma} = M_{\mathrm{h(l)}} + 2M_{\mathrm{bear}}$$

where M_{bear} is the friction moment in the bearing of the roll.

$$M_{\mathrm{bear}} = d_{\mathrm{rj}}\left(5000cd_{\mathrm{rj}} + 2.55 f_0 F_{\mathrm{h(l)}\Sigma}\right)$$

where c, f_0 are coefficients (for the radial spherical double-row roller bearings of the mills and calender rolls $c = 0.15$; $f_0 = 0.002$ [6]).

Then the total torque on the i - th roll can be written as

$$M_{i\Sigma} = \sum_j M_{\mathrm{h(l)}j} + 2M_{\mathrm{bear}}$$

where j is number of the roll gaps which forms given roll and adjacent to it rolls.

Capacity of a group - drive of rolls can be defined by the dependence

$$N_{\Sigma} = \frac{1}{\eta_{\mathrm{dr}}}\sum_{i=1}^{m} N_{i\Sigma}$$

where the capacity spent from the given roll on material deformation is defined by dependence

$$N_{i\Sigma} = M_{i\Sigma}\,\omega_i = M_{i\Sigma}\frac{W_i}{R_{\mathrm{r}}}$$

where W_i is linear velocity of the i- th roll, m/s;
η_{dr} is coefficient of efficiency of a group- drive of rolls which is defined as follows

$$\eta_{dr} = \eta_1 \eta_2 \eta_3 \eta_4 \eta_5^m$$

where are:

$\eta_1 = 0.99$ coefficient of efficiency of the electric motor;

$\eta_2 = 0.99$ coefficient of efficiency of sleeve and finger type coupling;

$\eta_3 = 0.90$ coefficient of efficiency of a block reducer;

$\eta_4 = 0.99$ coefficient of efficiency of a gear coupling;

$\eta_5 = 0.90$ coefficient of efficiency of an universal spindle.

Capacity of an individual drive of each roll of calender can be defined by the formula:

$$N_i = \frac{M_{i\Sigma}\,\omega_i}{\eta_{drr}} = \frac{M_{i\Sigma}\,W_i}{\eta_{drr} R_r}$$

Based upon the executed calculation of the general capacity, selection of the electric motor of a drive can be carried out.

4.9 Thermal calculation of the rolls

The equation of thermal and power balance of each calender is:

$$\Delta Q_m \pm Q_{ext} + Q_{diss} - Q_{loss} = 0$$

where ΔQ_m is change of the material enthalpy at passage by the inter-roll gap, W; Q_{ext} is thermal energy which needs to be brought to the roll (a sign «+») or to be taken away from it (a sign «–») in calendering process (an external system of thermal supplying of a roll), W; Q_{diss} is energy of dissipation received by the material as a result of irreversible deformation of shear in the inter-roll gaps which forms given roll and adjacent to it rolls, W; Q_{loss} is thermal losses in environment, W.
The dissipation component of power balance of each roll is defined by expression

$$Q_{diss} = \sum_{s=1}^{k} Q_{diss\,s} \;,$$

where $Q_{diss\,j}$ is dissipation capacity provided by the considered roll in a s- th inter-roll gap (from k inter-roll gaps which forms given roll and adjacent to it rolls) and defined by dependences [10]:

$$Q_{diss\ h} = \frac{KL\sqrt{2R_r h_{min}}}{h_{min}^n}\left(\frac{W_h}{2}\right)^{n+1}\left(\frac{1+2n}{n}\right)^n \frac{(n+2)}{3^{n+1}} \times$$
$$\times \int_{\xi_e}^{\xi_{in}} \int_0^1 \left| \frac{3(1+\psi)\left(\xi^2 - \xi_e^2\right)}{\left(1+\xi^2\right)^2}\varepsilon + \frac{1-\psi}{1+\xi^2} \right|^{n+1} \left(1+\xi^2\right) d\varepsilon\, d\xi;$$

$$Q_{\mathrm{diss\,l}} = \frac{KL\sqrt{2R_{\mathrm{r}}h_{\min}}}{h_{\min}^{n}}\left(\frac{W_{\mathrm{h}}}{2}\right)^{n+1}\left(\frac{1+2n}{n}\right)^{n}\frac{(n+2)}{3^{n+1}}\times$$

$$\times\int_{\xi_{\mathrm{e}}}^{\xi_{\mathrm{in}}}\int_{-1}^{0}\left|-\frac{3(1+\psi)\left(\xi^{2}-\xi_{\mathrm{e}}^{2}\right)}{\left(1+\xi^{2}\right)^{2}}\varepsilon+\frac{1-\psi}{1+\xi^{2}}\right|^{n+1}\left(1+\xi^{2}\right)d\varepsilon\, d\xi.$$

The value ΔQ_{m} can be defined as:

$$\Delta Q_{\mathrm{m}} = G\sum_{l} c_{p}\left(T_{\mathrm{e}}-T_{\mathrm{b}}\right)_{l} + \sum_{s} c_{p} G_{\mathrm{h(l)}}\left(T_{\mathrm{e}}-T_{\mathrm{b}}\right)_{s},$$

where T_{e} and T_{b} are final and initial temperature of the material at passage on the considered roll l-th zones located out of inter-roll gaps, and also s- th of inter-roll gaps which forms given roll and adjacent to it rolls (thus temperatures T_{e} and T_{b} are defined as averages on corresponding to half sections of the inter-roll gap from a considered roll), K; $G_{\mathrm{h(l)}}$ is the mass output provided by considered roll and equal mass flow rate through half s- th inter-roll gap from side of the considered roll; thus it is necessary to distinguish the mass output provided by high-velocity and low-velocity rolls, kg/s [6]:

$$G_{\mathrm{h}} = \frac{(3+\psi)}{4}G$$

$$G_{\mathrm{l}} = \frac{(3\psi+1)}{4}G$$

Thermal losses of each roll and also heat-carrier parameters in it are obtained according to a technique of parametrical and thermal calculation of mills for processing of plastic and rubber mixes [6].

5. CALCULATION EXAMPLE OF THE FOUR-ROLL "Z" CALENDER

Initial data

Drive of the rolls is individual

Type of the rolls: rolls with a peripheral channels

Radius of the roll barrel, m	0.355
Mass of the roll, kg	5500
Thickness of the product removed from the calender, m	0.0020
Width of the product removed from the calender, m	1.80
Velocity of removing product with the calender, m/s	0.60
Product removes from the 4 th roll at an angle 30° to horizon (configuration N 2 on Fig. 2)	
Friction factor in the 1 th inter-roll gap	1.20
Friction factor in the 2 th inter-roll gap	1.10
Friction factor in the 3 th inter-roll gap	1.00
Temperature of the 1 th roll, °C	150.0
Temperature of the 2 th roll, °C	154.0
Temperature of the 3 th roll, °C	158.0
Temperature of the 4 th roll, °C	162.0
Initial temperature of the material, °C	150.0

Maximal admissible temperature of a material, °C	180.0
The heat-carrier is water	
Rheological properties of a processed material:	
– power low index	0.200
– thermal coefficient of the rheological equation	8.10
– consistency index, Pa·s^n	135000.0
– temperature of definition of the consistency index, °C	150.0
Thermophysical properties of a processed material:	
– density, kg/m^3	1350.0
– mass thermal capacity, J/(kg·K)	1850.0
– thermal conductivity, W/(m·K)	0.175
Dimensionless coordinate of the end of the deformation zone of the inter-roll gap	0.40
Dimensionless coordinates of an input of the material in the inter-roll gaps:	
– in the 1 th inter-roll gap	3.0
– in the 2 th inter-roll gap	2.0
– in the 3 th inter-roll gap	2.0

Results of parametrical calculation of the calender

Linear velocities of working surfaces of the rolls, m/s (m/min):	
– velocity of the 1 th roll	0.45 (27.27)
– velocity of the 2 th roll	0.55 (32.73)
– velocity of the 3 th roll	0.60 (36.00)
– velocity of the 4 th roll	0.60 (36.00)
Values of the inter-roll gaps, m:	
– 1 th inter-roll gap	0.00206
– 2 th inter-roll gap	0.00180
– 3 th inter-roll gap	0.00172
Average temperature of the material in the inter-roll gaps, °C:	
– in the 1 th inter-roll gap	151.11
– in the 2 th inter-roll gap	152.96
– in the 3 th inter-roll gap	155.13
Average temperature of the material to an input in the inter-roll gaps, °C:	
– in the 1 th inter-roll gap	150.00
– in the 2 th inter-roll gap	151.28
– in the 3 th inter-roll gap	153.34

Distribution of temperature to an input in the 1 th inter-roll gap, °C:

150.00 150.00 150.00 150.00 150.00 150.00
150.00 150.00 150.00 150.00 150.00

Distribution of temperature to an input in the 2 th inter-roll gap, °C:

154.00 152.87 152.00 151.42 151.05 150.89
150.95 151.10 151.03 150.29 148.50

Distribution of temperature to an input in the 3 th inter-roll gap, °C:

158.00 155.55 153.80 152.76 152.19 152.04
152.29 152.78 153.14 152.84 151.34

Average temperature of the material on an output of the inter-roll gaps, °C:	
– 1 th inter-roll gap	151.47
– 2 th inter-roll gap	153.33
– 3 th inter-roll gap	155.50

Distribution of temperature on an output of the 1 th inter-roll gap, °C:

150.00 152.15 151.83 151.27 150.66 150.46
150.80 151.25 151.63 152.10 154.00

Distribution of temperature on an output of the 2 th inter-roll gap, °C:

154.00 155.05 153.92 152.78 151.75 151.20
151.70 152.41 152.80 153.07 158.00

Distribution of temperature on an output of the 3 th inter-roll gap, °C:

158.00 157.63 155.69 154.14 152.92 152.20
152.99 154.10 154.97 155.83 162.00

Distribution of temperature at removal of the product from the calender, °C:

162.00 159.11 156.76 155.12 154.15 153.78
153.92 154.36 154.68 154.35 152.81

Roll-separating forces in the inter-roll gaps:

– specific (full) in the 1 th inter-roll gap (kN/m (kN))	217 (391)
– specific (full) in the 2 th inter-roll gap (kN/m (kN))	222 (400)
– specific (full) in the 3 th inter-roll gap (kN/m (kN))	217 (391)

Specific (full) forces operating on the rolls, kN/m (kN):

– on the 1 th roll	220 (396)
– on the 2 th roll	303 (545)
– on the 3 th roll	334 (601)
– on the 4 th roll	219 (395)

The capacity brought to the rolls, kW:

– to the 1 th roll	18.6
– to the 2 th roll	34.3
– to the 3 th roll	24.6
– to the 4 th roll	11.7

Capacity of driver engines at coefficient of efficiency 0.770, kW:

– of the 1 th roll	24.1
– of the 2 th roll	44.5
– of the 3 th roll	31.9
– of the 4 th roll	15.2

Results of thermal calculation of the calender

Volume flow rate of the heat-carrier in the roll, m^3/s:

– in the 1 th roll	0.129E-02
– in the 2 th roll	0.242E-02
– in the 3 th roll	0.334E-02
– in the 4 th roll	0.294E-02

Stream speed of the heat-carrier in the peripheral channels of the roll, m/s:

– in the 1 th roll	0.056
– in the 2 th roll	0.105
– in the 3 th roll	0.145
– in the 4 th roll	0.127

Average temperature of the heat-carrier in the peripheral channels the rolls, °C:

– in the 1 th roll	158.40
– in the 2 th roll	167.50
– in the 3 th roll	175.56
– in the 4 th roll	177.86

The energy brought to the roll from system of a heat supply, kW:

– to the 1 th roll	10.21
– to the 2 th roll	19.04
– to the 3 th roll	26.20
– to the 4 th roll	23.01

Difference of temperatures at the surface of the roll peripheral channels and at the work surface of the roll (roll periphery), °C:

– of the 1 th roll	+5.43
– of the 2 th roll	+10.13
– of the 3 th roll	+13.94
– of the 4 th roll	+12.25

6. CONCLUSION

The presented technique has shown the efficiency at designing and commercial maintenance of three-roll and four-roll calenders produced by factory “Bolshevik” (Kyiv, Ukraine) with the rolls 550×1500 mm, 610×1800 mm, 710×1800 mm, 950×2800 mm. The developed technique allows obtaining (calculate) main parameters of calendering process of thermoplastic polymeric materials.

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Nomenclature

A, B	design complexes;
c	design coefficient;
c_p	mass thermal capacity, J/(kg·K);
d	diameter, m;
f_0	design coefficient;
F	roll-separating force, N;
G	mass flow rate, kg/s;
h	half of size of the inter-roll gap, m;
i	order number of the roll;
j	order number of the roll gap;
k	quantity of the roll gaps which forms given roll and adjacent to it rolls;
K	consistency index, Pa·s^n;
L	width of a product which is removed from the calender, m;
m	quantity of the calender rolls;
M	torque, N·m;
n	power low index;
N	capacity, W;
p	pressure, Pa;
P	tangent force which operate on to the roll from the processed material, N;
q	surface heat flow, W/m^2;
$q_{\rm diss}$	intensity of dissipation energy, W/m^3;
Q	heat flow, W;
r	coordinate directed along radius of the roll, m;
R	radius, m;
s	order number of the inter-roll gap which forms given roll and adjacent to it rolls;
S	ares, m$^{2;}$
t	time, s;
T	temperature, K;
w	linear velocity of the material, m/s;
W	velocity of the roll periphery or velocity of removing of the product from the calender, m/s;
x, y	Cartesian coordinates;
α	heat transfer coefficient, W/(m^2·K); an angle (in the presence of an index), rad;
β	thermal coefficient of the rheological equation; angle of application of force F or P to the roll (in the presence of an index), rad;
γ	the central angle of working zone of the roll, rad;
δ	thickness of a layer of the material outside of the inter-roll gap, m;
ε	dimensionless analogue of coordinate y;
η	coefficient of efficiency;
λ	thermal conductivity, W/(m·K);
ξ	dimensionless analogue of coordinate x;
ρ	density, kg/m^3;
τ	tangential stresses, Pa;
φ	angle of slope of an axis of a overhanging roll, rad;
ψ	friction factor in the inter-roll gap;
ζ	angle of removing of the product from the calender concerning a horizontal, rad;
ω	angular velocity of the roll, rad/s.

TEHNIKA ZA PARAMETARSKO I TERMALNO PRORAČUNAVANJE KALANDERA ZA PROCESIRANJE PLASTIČNIH I GUMENIH MEŠAVINA

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REZIME

U ovom radu je prikazana tehnika parametarskog i termalnog proračuna kalandera za procesiranje power-low modela fluida. Ova tehnika je pogodna za analizu procesiranja kalandera različitih tipova sa rolnama identičnih prečnika, trenja unutar rolne, i takođe sekventnog pomeranja materijala na kalanderu. Elaboriran je primer proračuna "Z" kalandera sa četiri rolne.

Ključne reči: *calander, parametri procesa, tehnika parametarskog i termalnog proračuna*